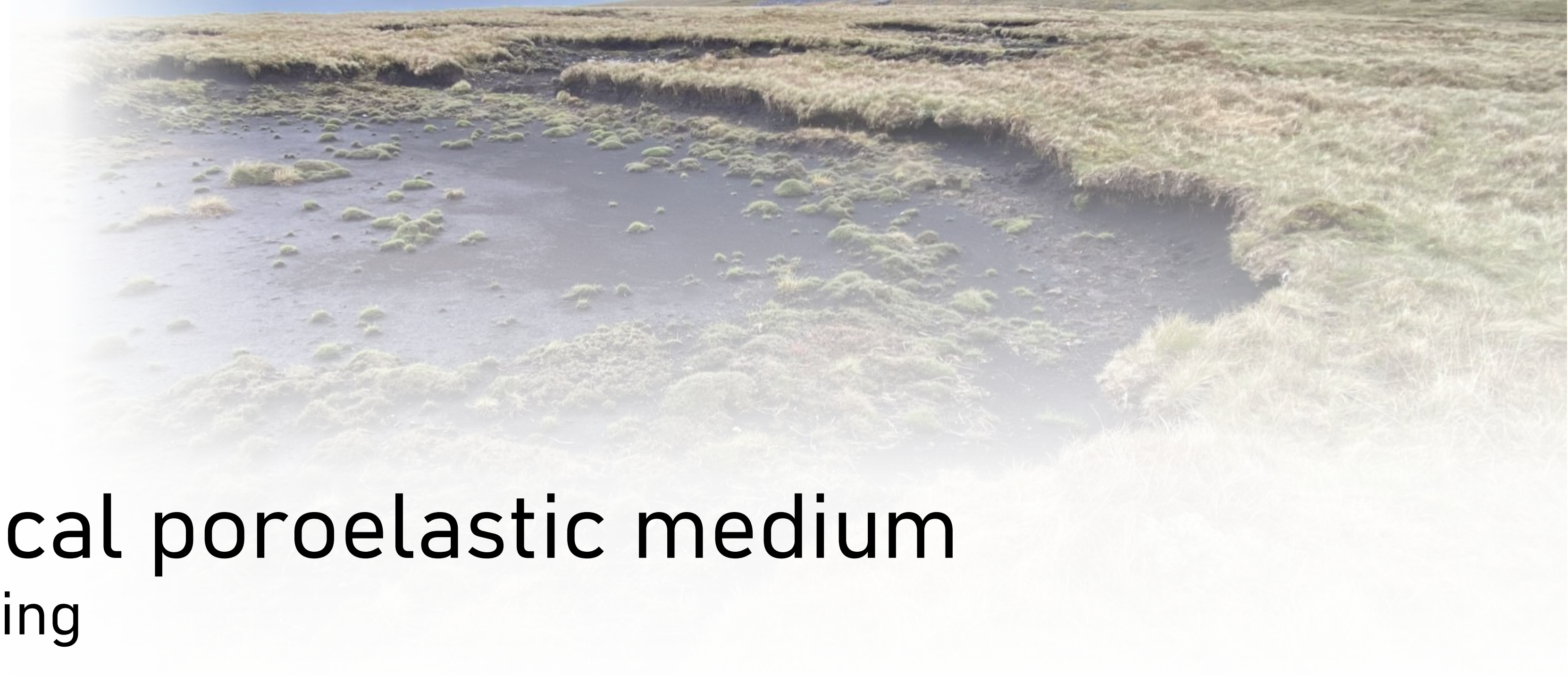
Mathematical Modelling for Peat's Sake
University of Warwick
Tuesday 9th June 2026, 14:50

image credit: Alan Hughes, geograph.org.uk



Peat as a hierarchical poroelastic medium
some ideas from gel modelling

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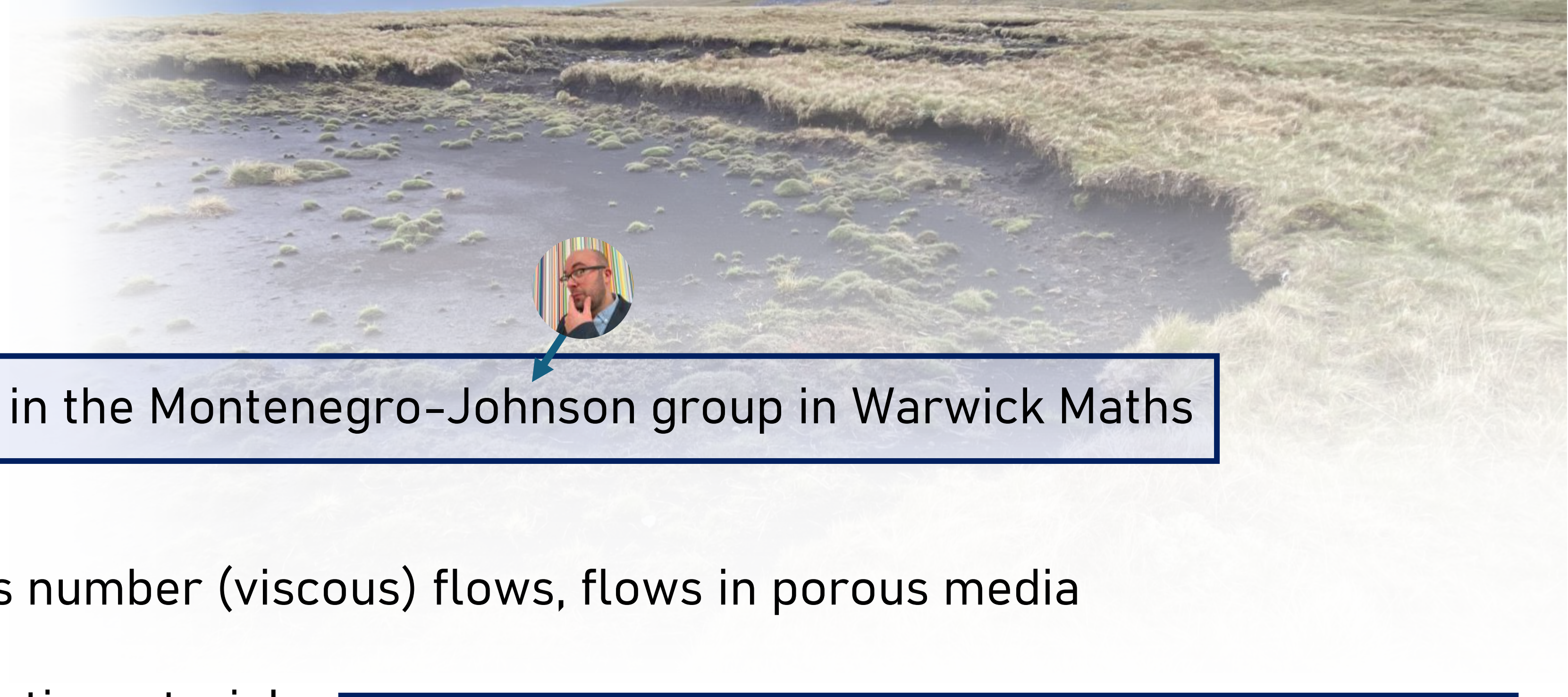

Applied mathematician – postdoc in the Montenegro-Johnson group in Warwick Maths

- Fluid mechanics: low-Reynolds number (viscous) flows, flows in porous media
- Solid mechanics: nonlinear elastic materials
- Geo(physical/logical/…) applications

launched at a MiRaW day 09/12/2024

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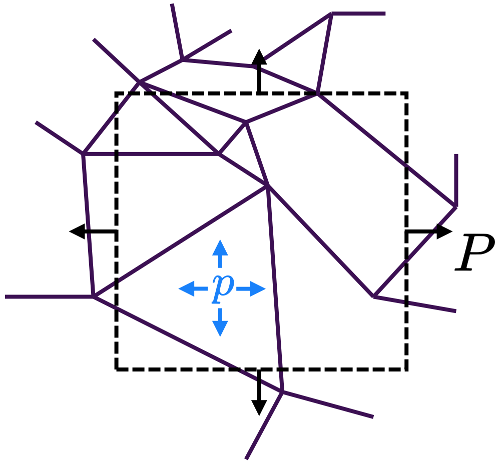
sites.google.com/view/ukhydrogels/

Who am I?
(the existential slide)

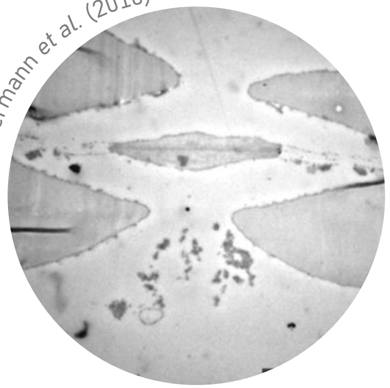
Models for hydrogels

hydrogels = **crosslinked hydrophilic scaffold** surrounded by **adsorbed water molecules**



- **Solid component** (polymer) and **liquid component** (water)
- Elastic network must **stretch to accommodate water** in swelling
- **Dries out** to form a brittle solid polymer
- Support **interstitial flows** (can act as filters, etc.)

Royal Horticultural Society



Pittermann et al. (2010)

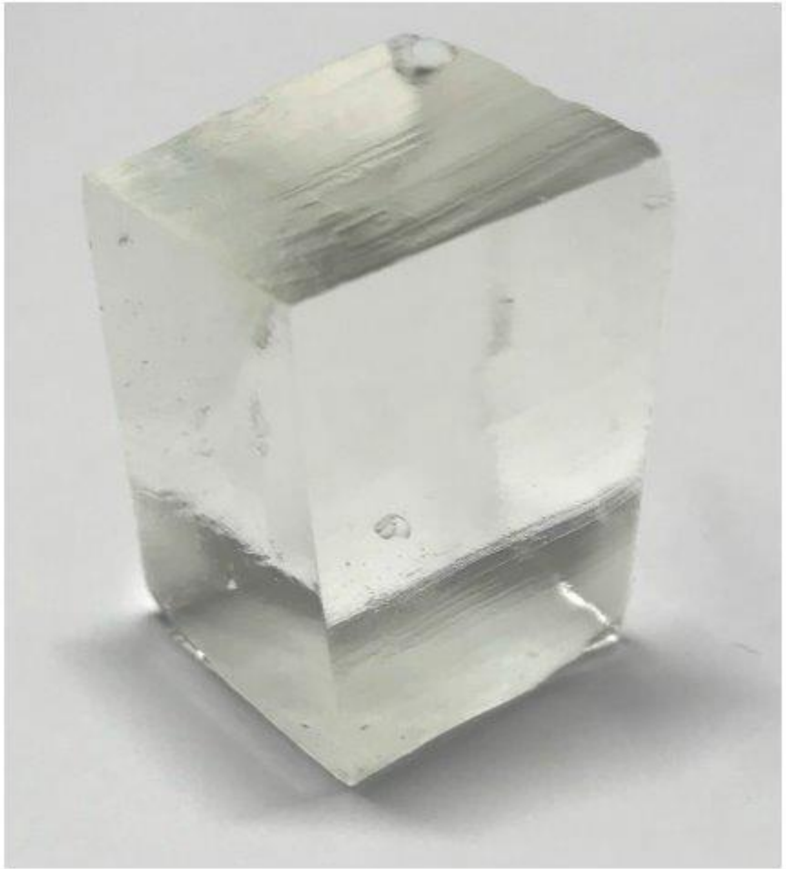


image credit: Julian Paren, geograph.org.uk

Models for peat?

hydrogels = **crosslinked hydrophilic scaffold** surrounded by **adsorbed water molecules**

- **Solid component** (organic matter) and **liquid component** (water)
- Volume expands **to fit water** – <u>cellulose is hydrophilic</u>
- **Dries out** to form a brittle crumbly 'scaffold'
- Support **interstitial flows** (rainfall can filter through)

image credit: Antonio Jordán, imaggeo.egu.eu

We have a lot of mathematical models for hydrogels that describe these materials well. Can we use them for peat too?

Models for peat?

hydrogels = **crosslinked hydrophilic scaffold** surrounded by **adsorbed water molecules**

There are two schools of thought on how to model hydrogels:

Energy-based approaches

$$\mathcal{W} = \underbrace{\frac{k_B T}{2\Omega_p}\left[\mathrm{tr}\left(\mathbf{F_d}\,\mathbf{F_d^T}\right) - 3 + 2\log\phi\right]}_{\text{Gaussian-chain elasticity}} + \underbrace{\frac{k_B T}{\Omega_f}\left[\frac{1-\phi}{\phi}\log(1-\phi) + \chi(\phi,\,T)(1-\phi)\right]}_{\text{Mixing of polymer and water}}$$

Poroelastic models *(see Matteo's talk)*

- Solid and liquid phase; both exert stress on each other
- Elasticity and osmotic effects balance fluid pressures
- Fluid flows are driven by gradients in fluid pressure

image credit: Antonio Jordán, imaggeo.egu.eu

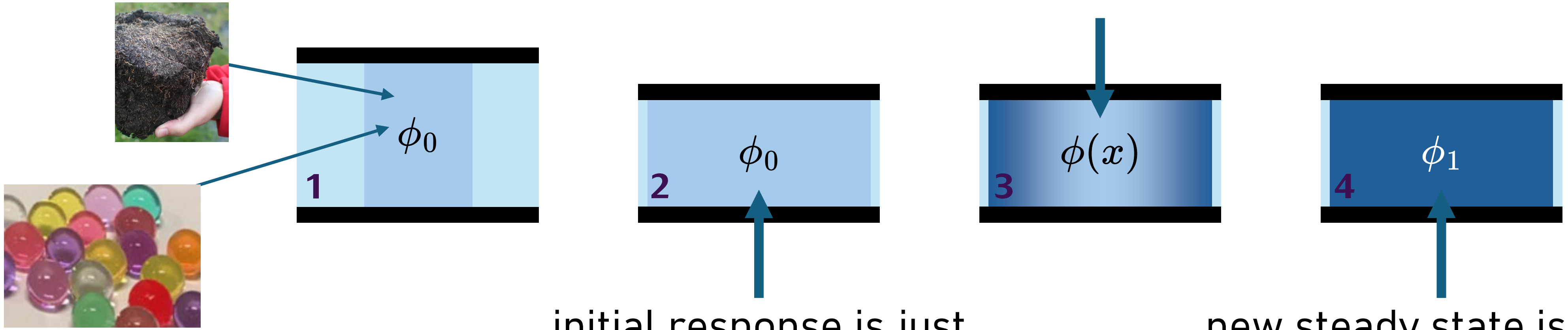
Mechanical characterisation

ϕ = solid fraction

water is slowly driven out
through the pore spaces

gives a permeability

independent of gel chemistry!



swollen material
surrounded by water;
compress between
impermeable plates

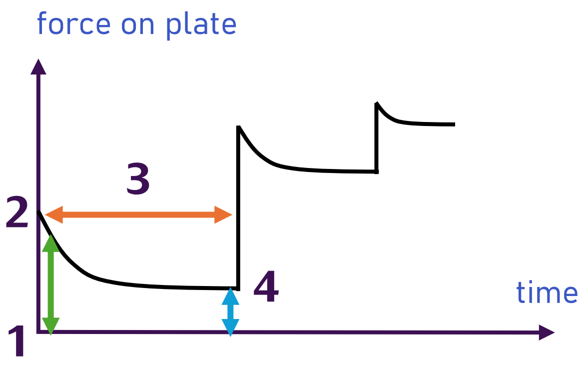
initial response is just
elastic and doesn't involve
volumetric change.

gives a [shear] 'stiffness'

new steady state is drier,
with force from plates
balancing desire to draw in
more water

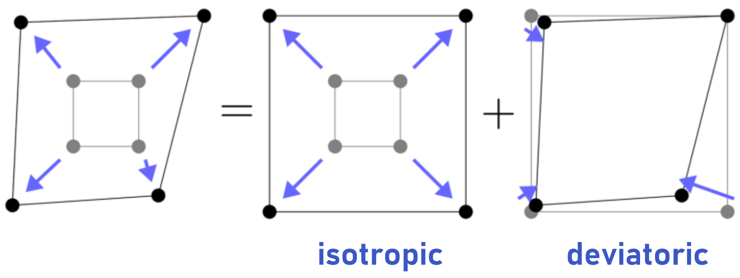
gives an osmotic pressure

Mechanical characterisation



The **shear modulus**, **permeability** and **osmotic pressure** will all change with solid fraction, and the way that this happens can be deduced by repeating the experiment

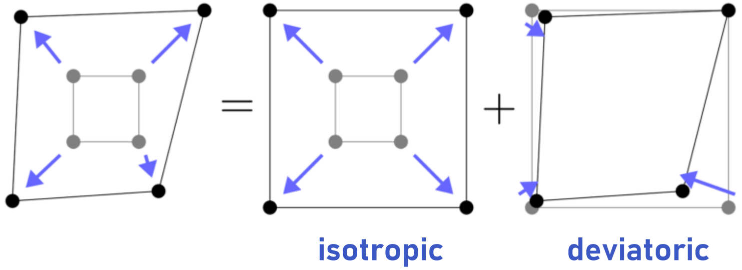
Add in the assumption that deformation is mostly just due to expansion/contraction (small shear)



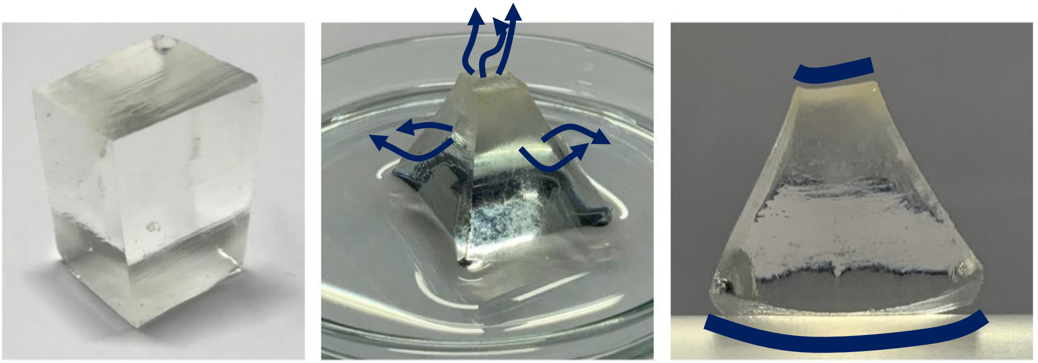
$$\boxed{\frac{\partial \phi}{\partial t} + \boldsymbol{q} \cdot \boldsymbol{\nabla} \phi = \boldsymbol{\nabla} \cdot \left\{ \frac{k(\phi)}{\mu_l} \left[\phi \frac{\partial \Pi}{\partial \phi} + \frac{4\mu_s(\phi)}{3} \left(\frac{\phi}{\phi_0} \right)^{1/3} \right] \boldsymbol{\nabla} \phi \right\}}$$

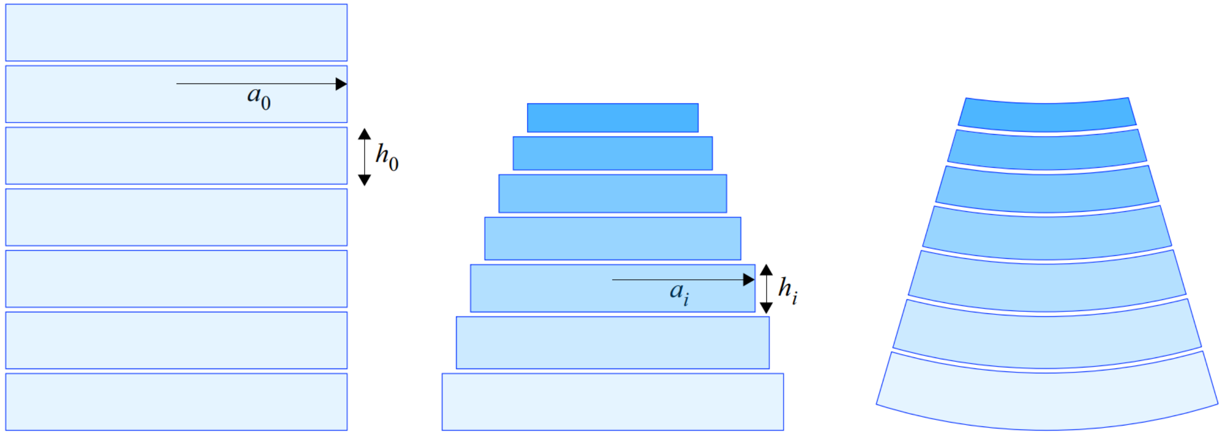
Webber & Worster (J. Fluid Mech. 2023); Webber, Etzold & Worster (J. Fluid Mech. 2023)

Mechanical characterisation

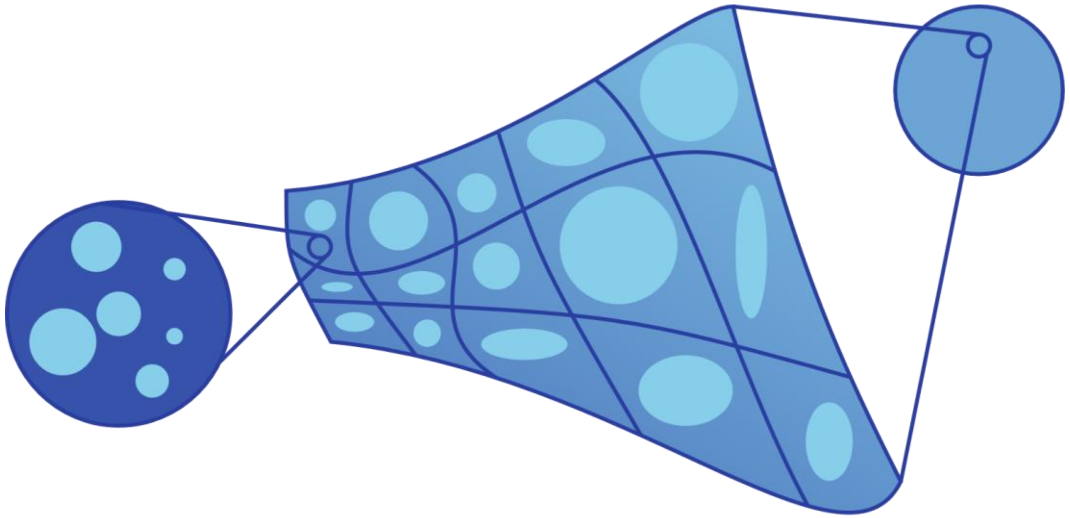


$$\boxed{\frac{\partial \phi}{\partial t} + \boldsymbol{q}\cdot\boldsymbol{\nabla}\phi = \boldsymbol{\nabla}\cdot\left\{\frac{k(\phi)}{\mu_l}\left[\phi\frac{\partial \Pi}{\partial \phi} + \frac{4\mu_s(\phi)}{3}\left(\frac{\phi}{\phi_0}\right)^{1/3}\right]\boldsymbol{\nabla}\phi\right\}}$$





Hierarchical poroelastic materials (HPEMs)



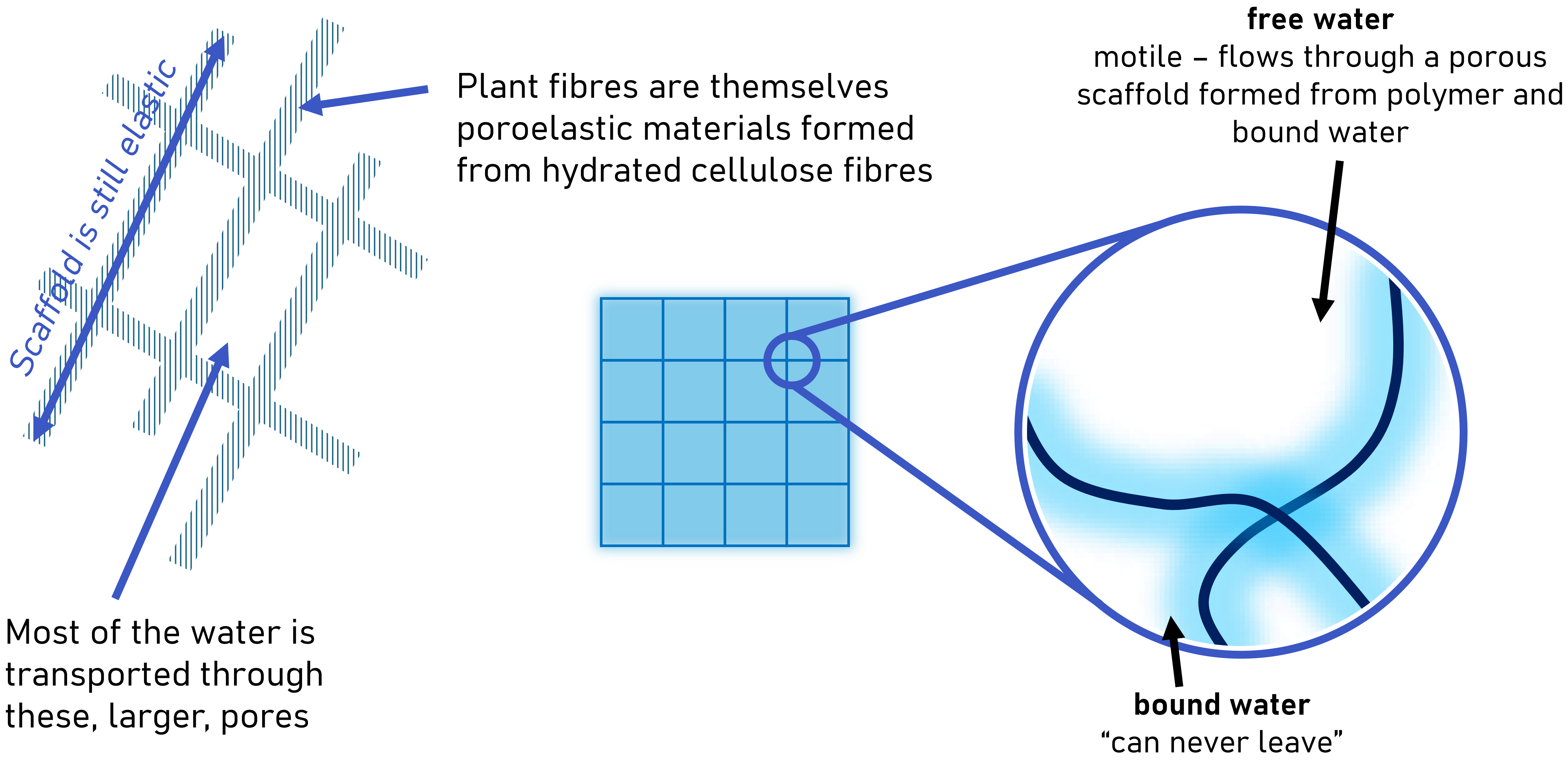
Hierarchical poroelastic materials (HPEMs)

- have a **regular microstructure** built from a poroelastic base

- derive their **bulk properties** from this structure (are metamaterials)

- have a **microstructure that changes** in time and space

Hierarchical poroelastic materials (HPEMs)



Hierarchical poroelastic materials (HPEMs)

Multiple lengthscales

To numerically model these materials, we need a fine spatial resolution, so numerical simulation is intractable

*requires **asymptotic homogenisation***

Evolving microstructure

The microstructure can evolve in time rapidly owing to swelling and deswelling, breaking homogenisation

*requires **new homogenised models***

- The microstructure might have sharp interfaces
- The properties can change irreversibly (fracture, etc.)
- HPEMs interface with their environment; what are the boundary conditions?

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What I do

- Poroelastic modelling
- Models that don't depend on a detailed understanding of the microscale physics
- Interactions with environmental processes (especially freezing)

Questions to answer

- Can we characterise peat in the way detailed here?
- What are the effects of anisotropy on material properties and transport?
- Can we model HPEMs?
- Which boundary conditions?

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