Fluid Dynamics Research Centre seminar
University of Warwick
Wednesday 4th November 2026, 15:00

Soft solids meet hard flow

pressure-driven fluid flow through deformable porous materials

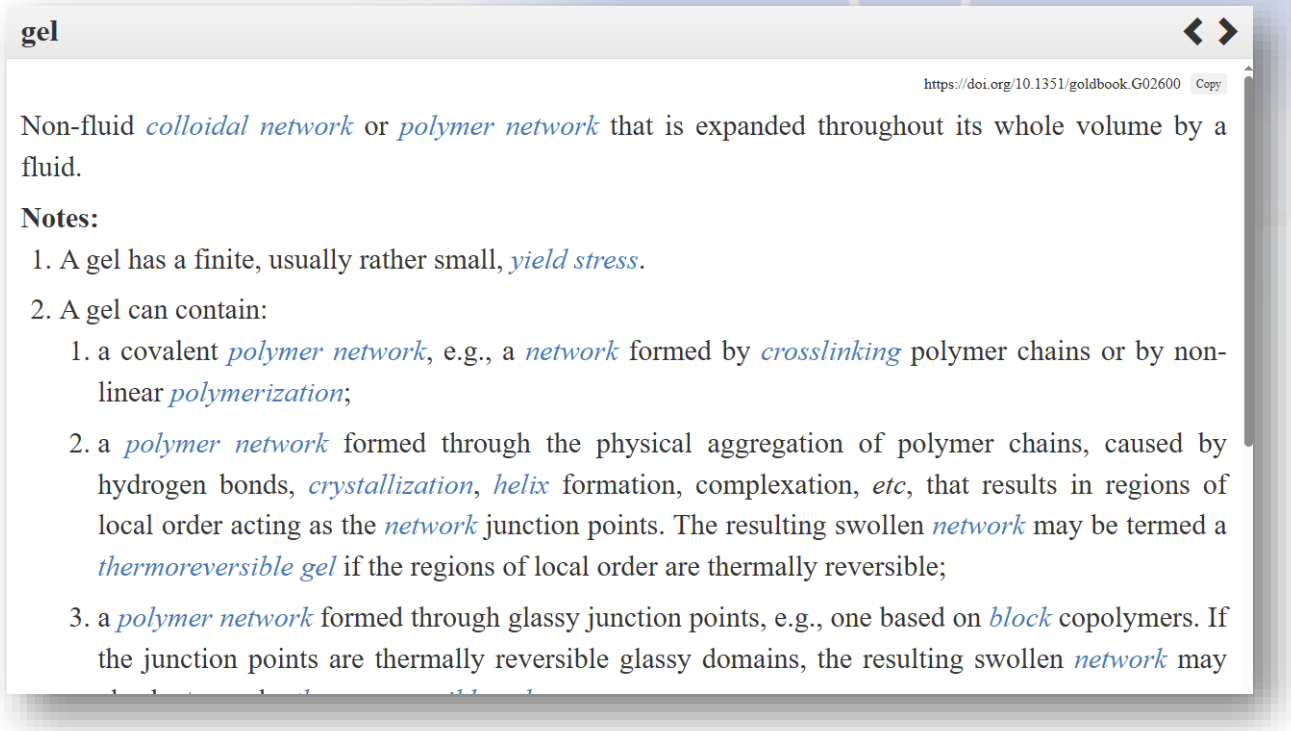
Joseph Webber
Mathematics Institute, University of Warwick

joe.webber@warwick.ac.uk
jwebber.github.io

UNIVERSITY OF WARWICK

What's a gel?

Chemists can't decide, but the IUPAC definition is



gel

https://doi.org/10.1351/goldbook.G02600 Copy

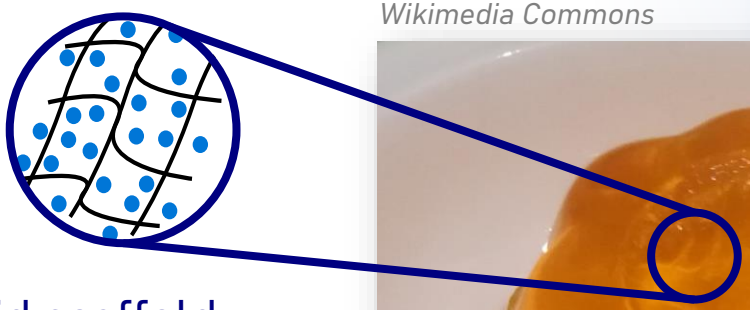
Non-fluid *colloidal network* or *polymer network* that is expanded throughout its whole volume by a fluid.

Notes:

1. A gel has a finite, usually rather small, *yield stress*.
2. A gel can contain:
    1. a covalent *polymer network*, e.g., a *network* formed by *crosslinking* polymer chains or by non-linear *polymerization*;
    2. a *polymer network* formed through the physical aggregation of polymer chains, caused by hydrogen bonds, *crystallization*, *helix* formation, complexation, *etc*, that results in regions of local order acting as the *network* junction points. The resulting swollen *network* may be termed a *thermoreversible gel* if the regions of local order are thermally reversible;
    3. a *polymer network* formed through glassy junction points, e.g., one based on *block* copolymers. If the junction points are thermally reversible glassy domains, the resulting swollen *network* may

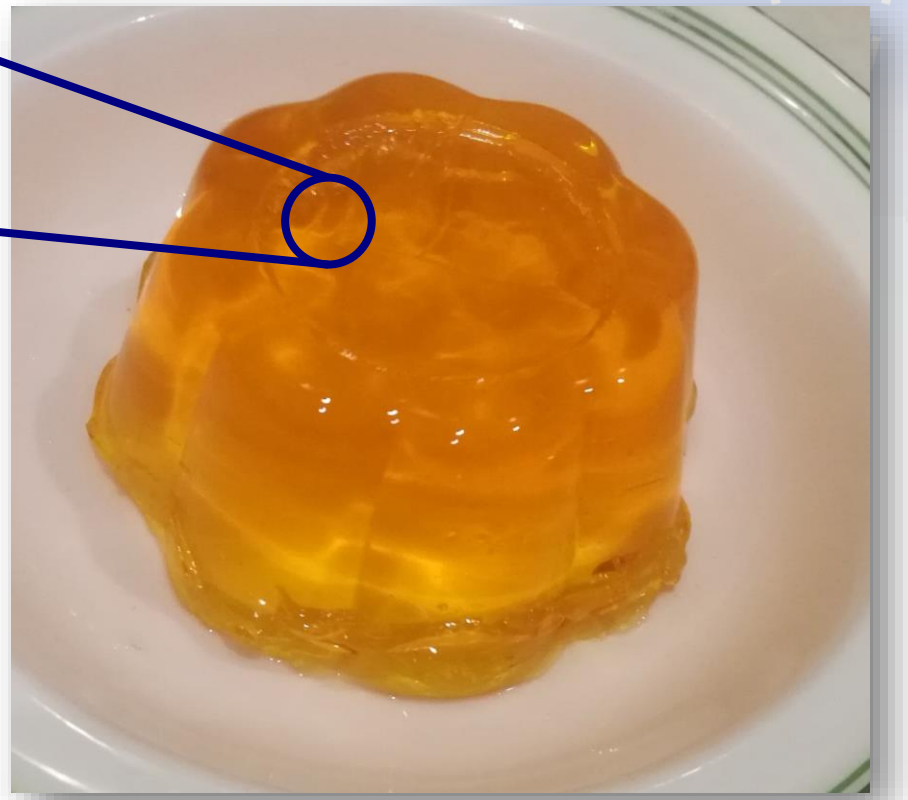
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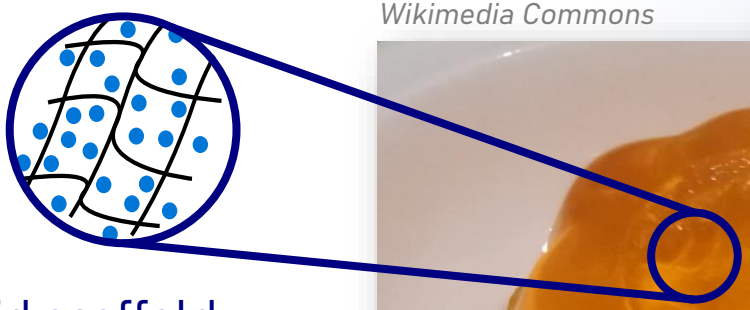
solid scaffold
surrounded by at least
some free water

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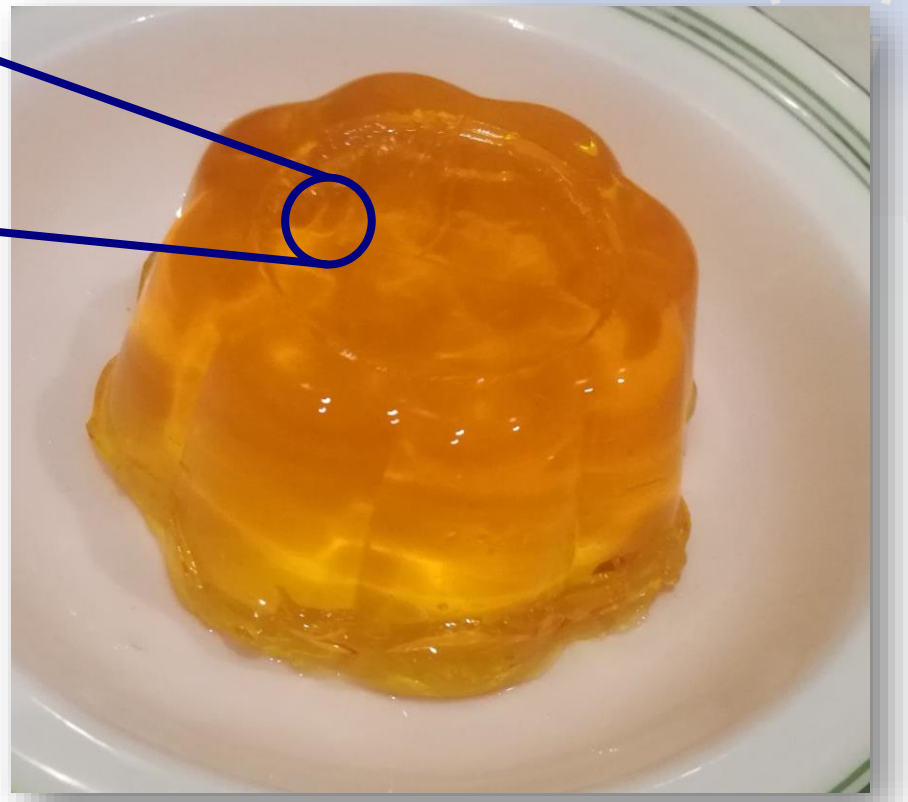


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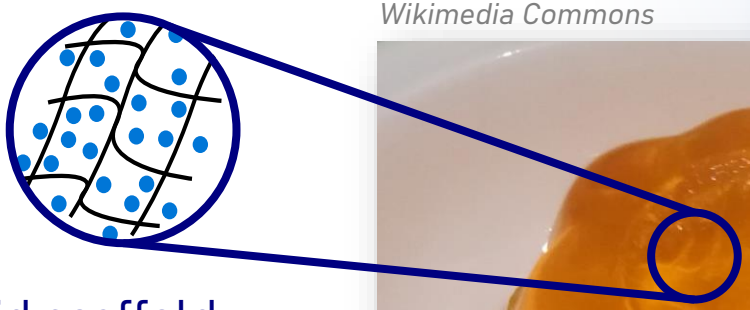
in a hydrogel, a lot of the water is "bound" – but some is motile!

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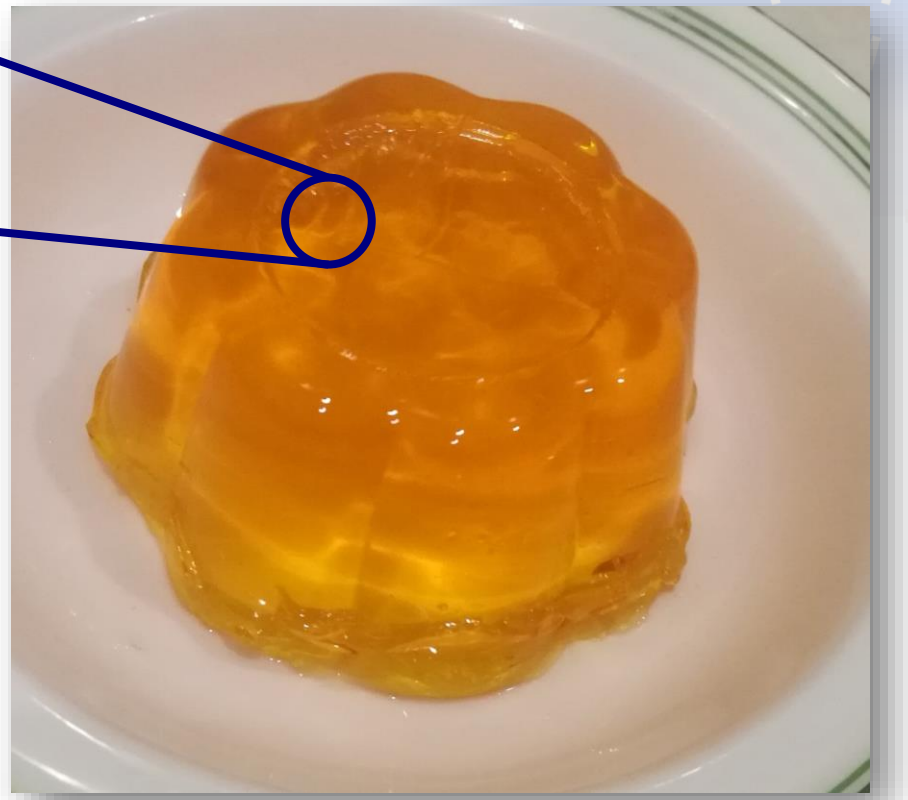


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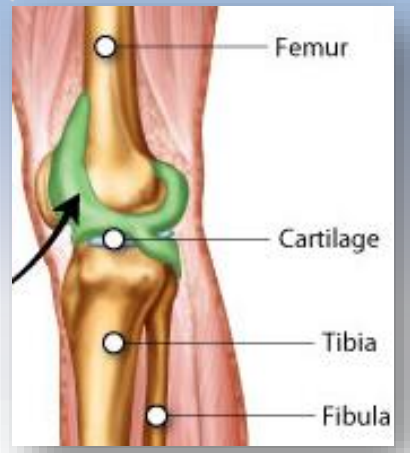


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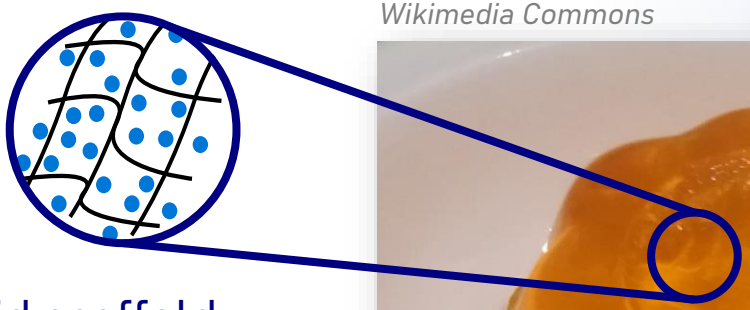


labxchange.org



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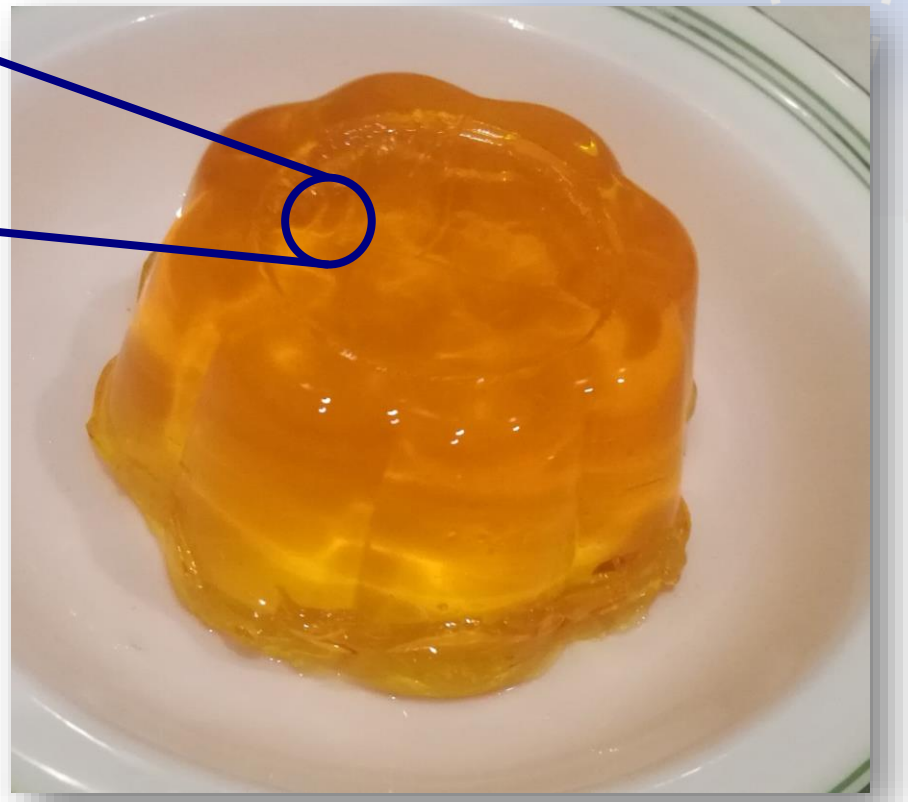


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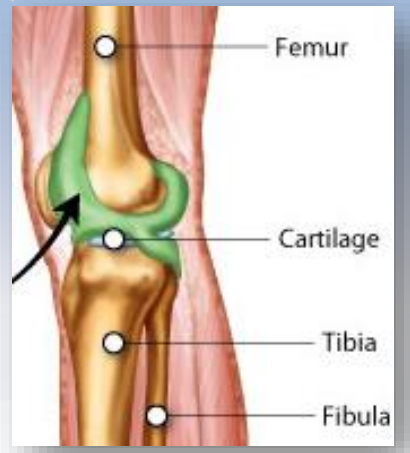


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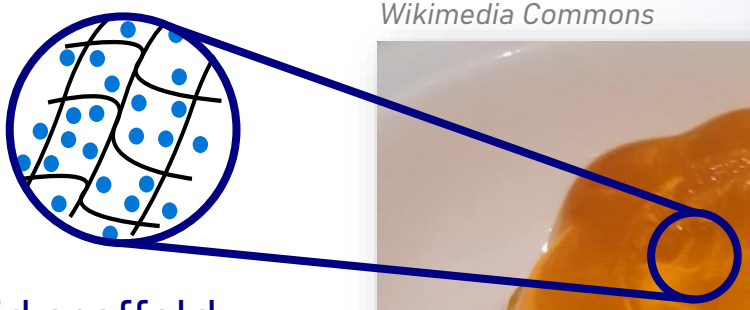


Shankar S. (Flickr)



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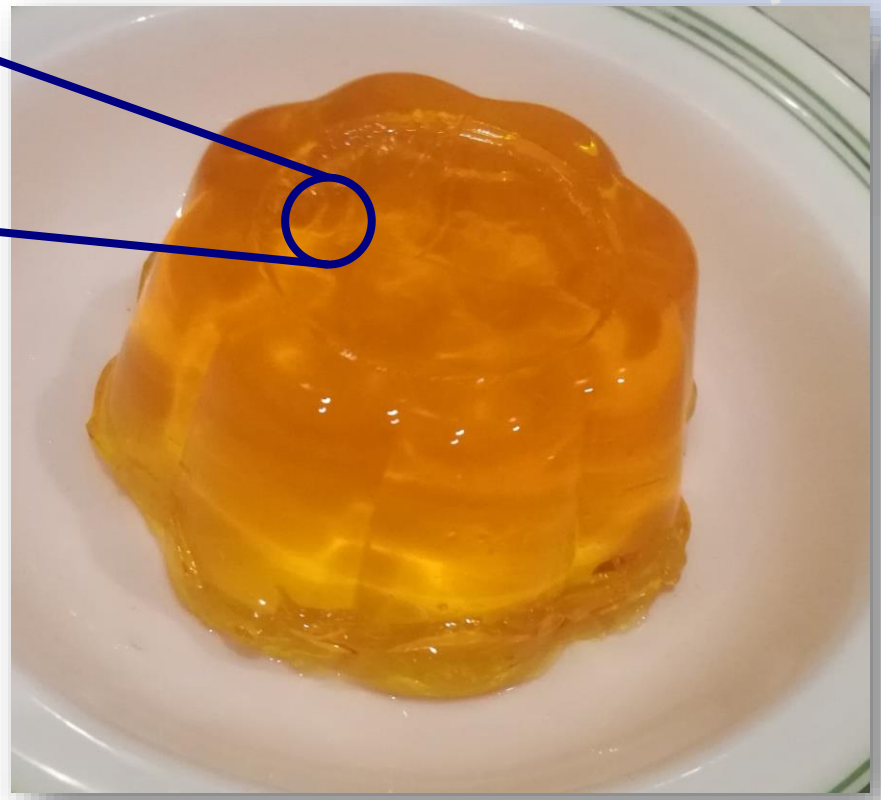


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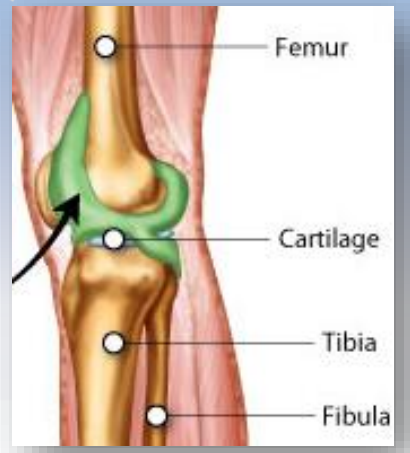


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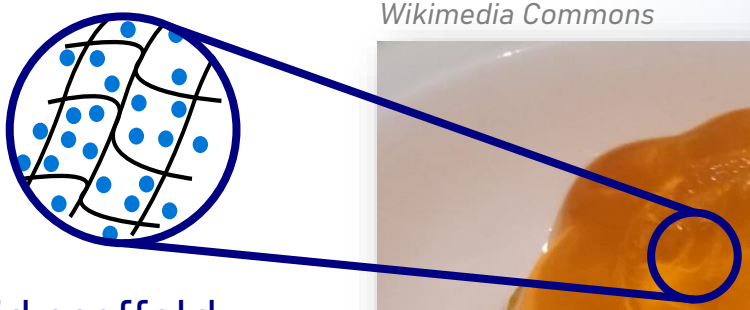


reddit.com/r/espresso



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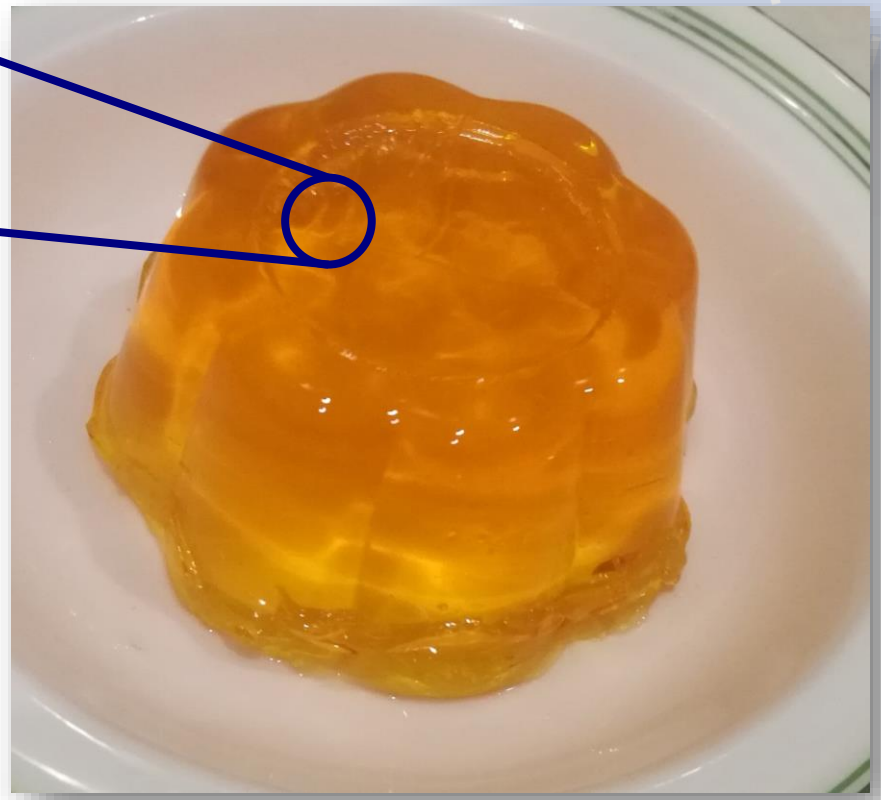


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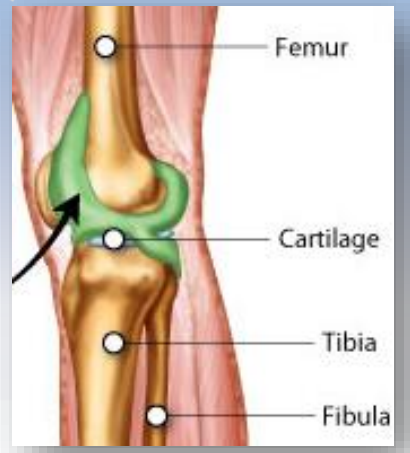


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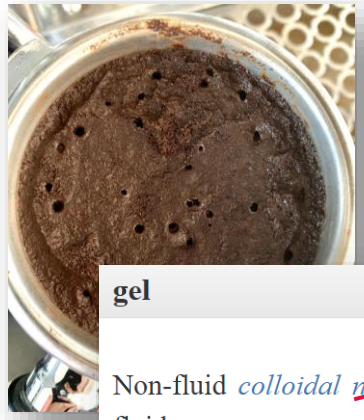
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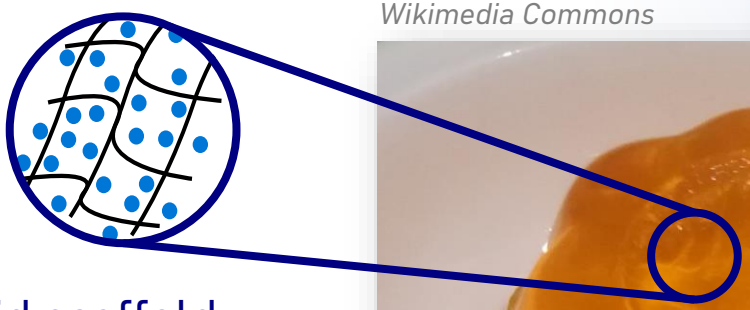


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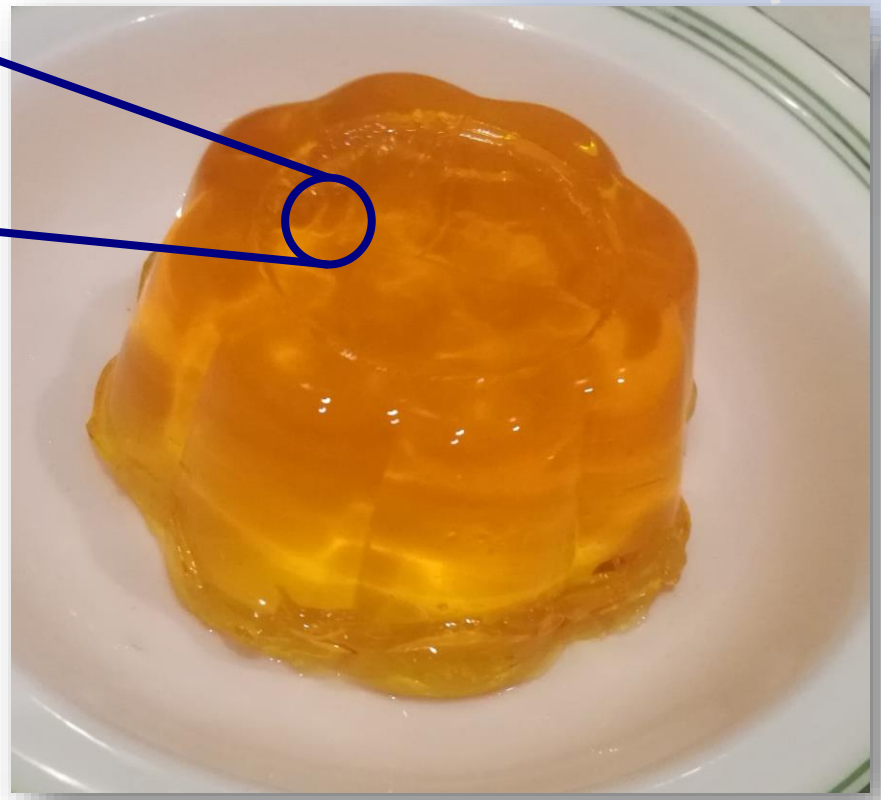


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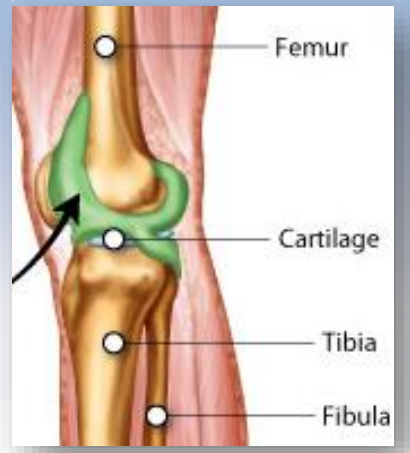


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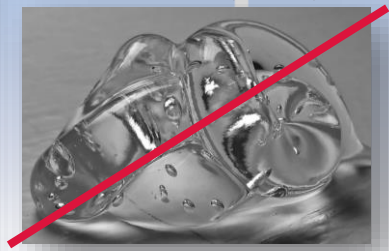
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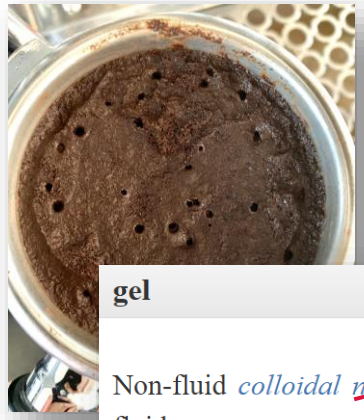
Steve Johnson (Flickr)



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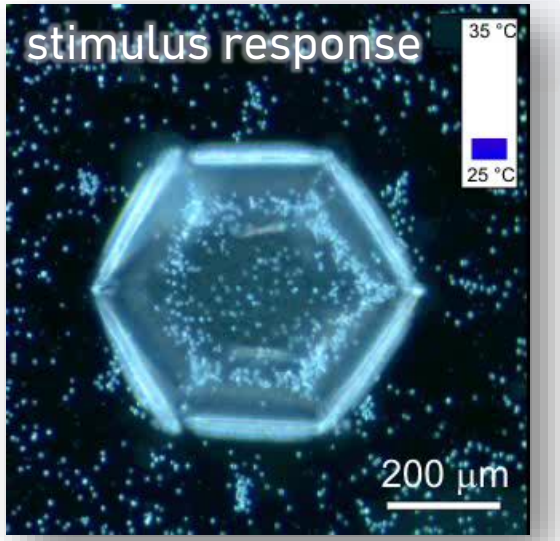
Gels + flow

Traditionally, flows in porous, elastic, materials are slow and occur at the same rate as the slow reconfiguration of the matrix in the absence of significant external pressures

Tim Dowling / The Guardian



Stoychev et al. – *Soft Matt.* ***7*** *(2011)*



Grae Worster



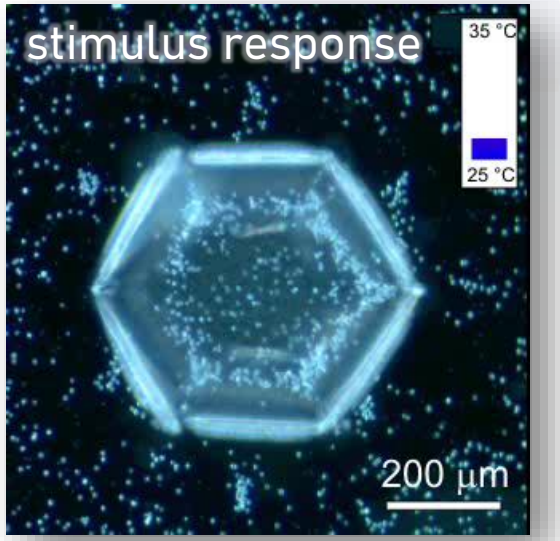
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Pressure-driven flows can be important!

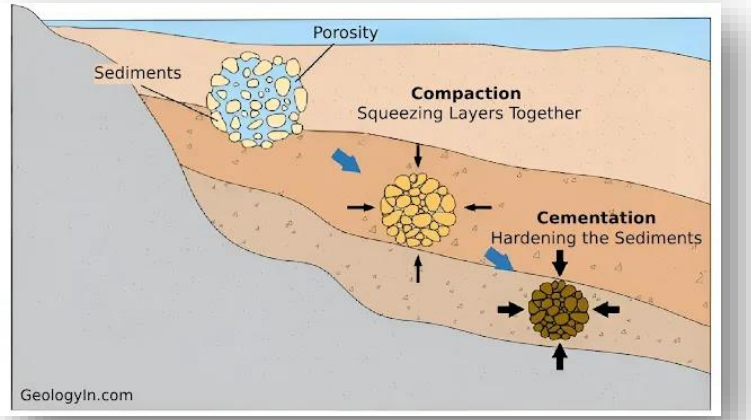
- Fluid flow in sedimentary basins
- Squeezing water through paper fibres
- Water flow in intestinal boluses
- Shock absorbers in the knee

- Coffee brewing?

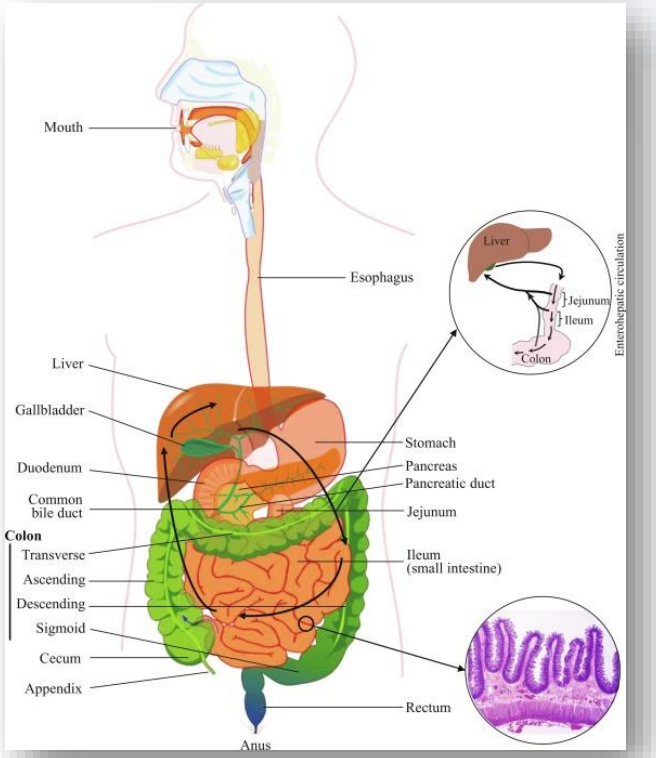
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GeologyIn



Reference Module in Biomedical Sciences (2018), ed. S. A. Saghir (Elsevier)

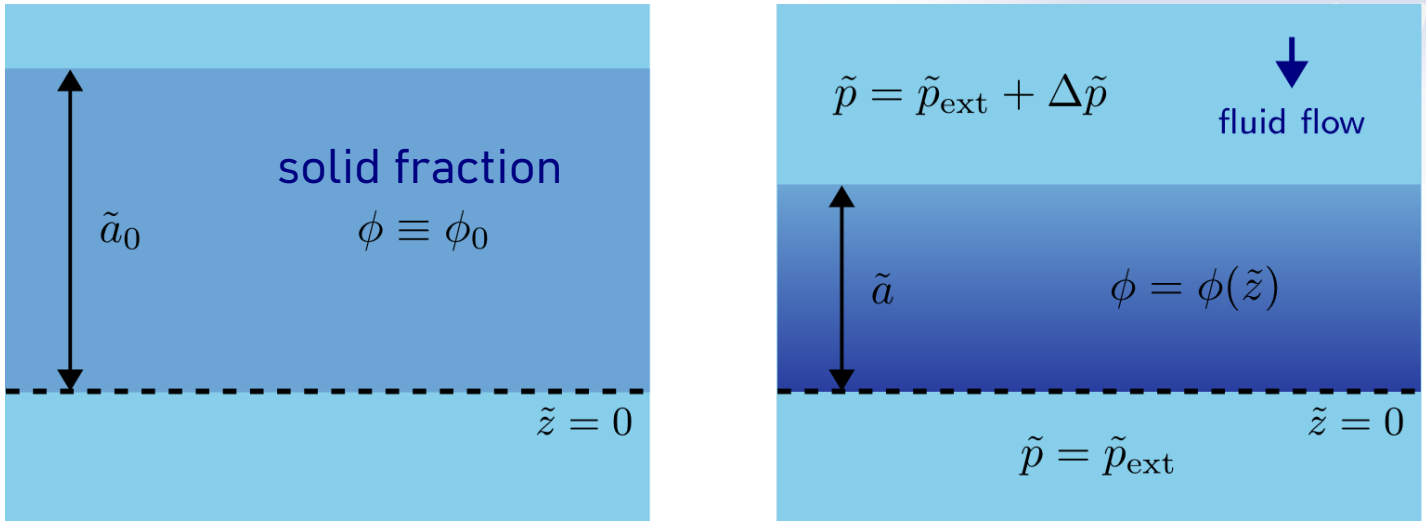


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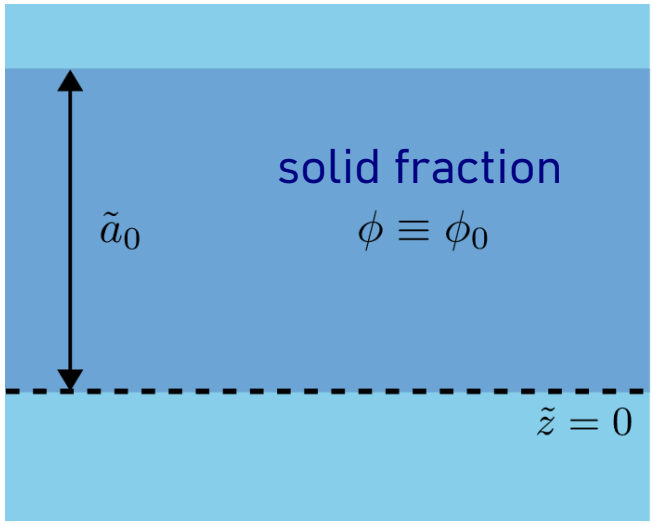
base sits on a solid, perfectly permeable mesh

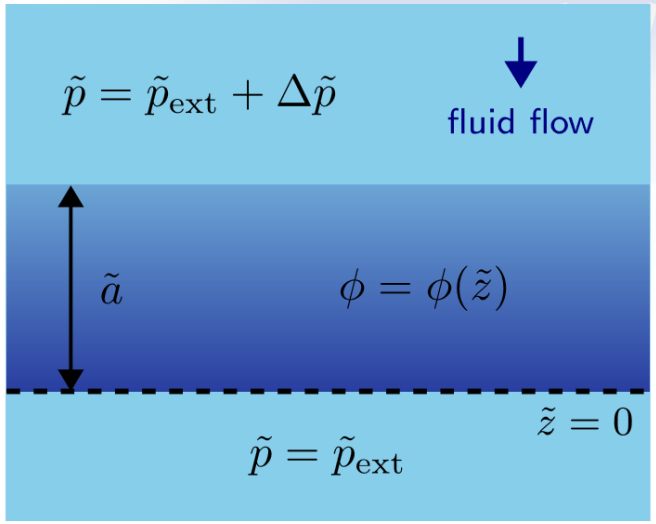
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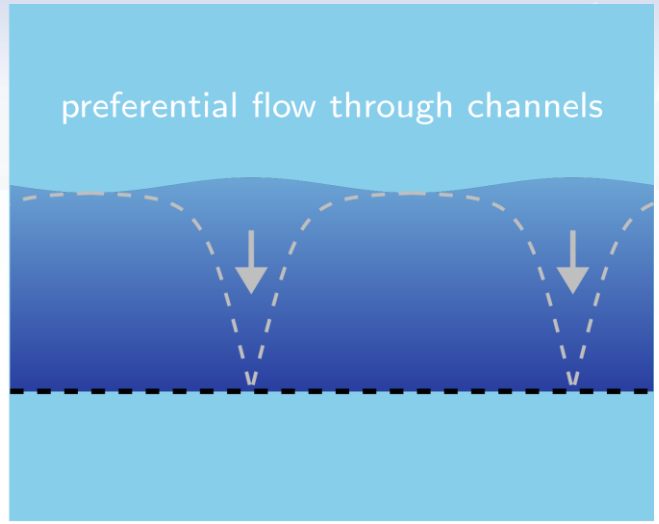
supports arbitrary stresses
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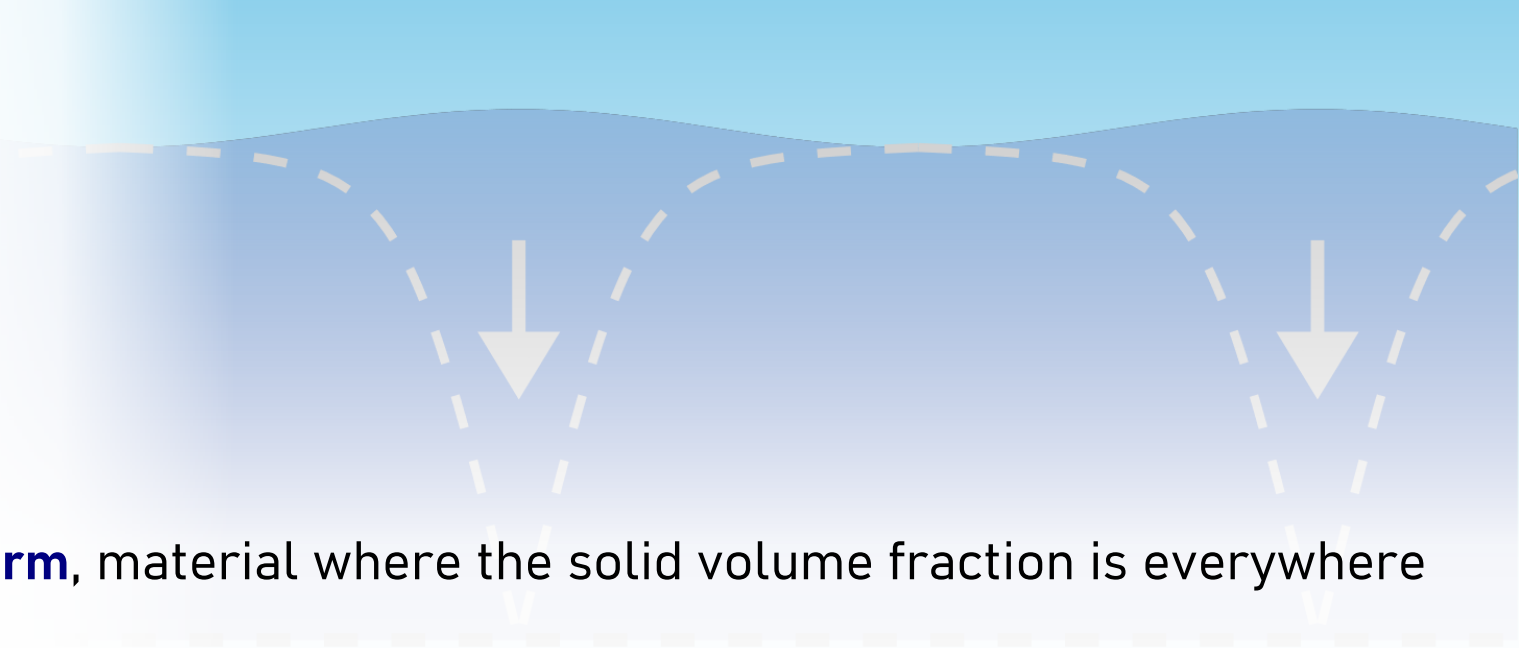




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Pressure and stress

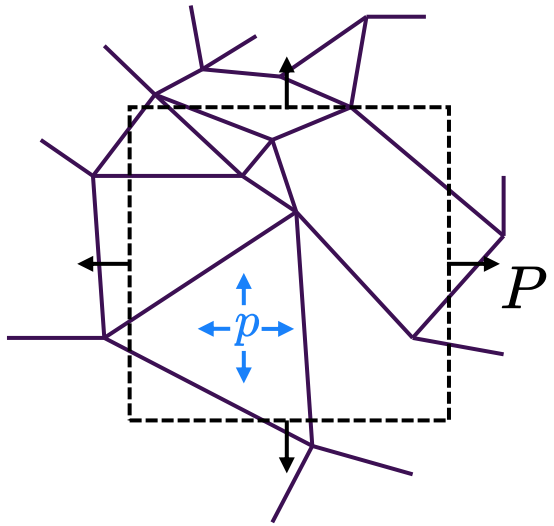


Take, as a base state, an **isotropic**, **uniform**, material where the solid volume fraction is everywhere

$$\phi \equiv \phi_0$$

This will depend on temperature, chemical stimuli, electrostatics, chemistry, … but we ignore all of that for today.

Pressure and stress



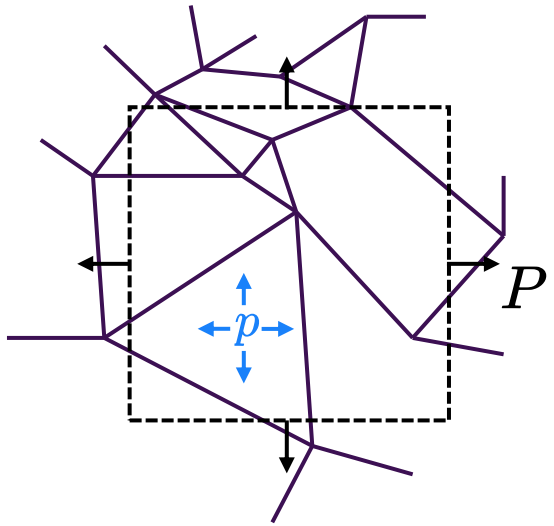
Like normal, we define a stress tensor such that the force per unit area across a surface with normal ***n*** is given by

$$\boldsymbol{\Sigma} = \boldsymbol{\sigma} \cdot \mathbf{n}$$

This stress comes from both the matrix (elasticity etc.) and the fluid

$$\boldsymbol{\sigma} = \boldsymbol{\sigma}^{(e)} + \boldsymbol{\sigma}^{(f)}$$

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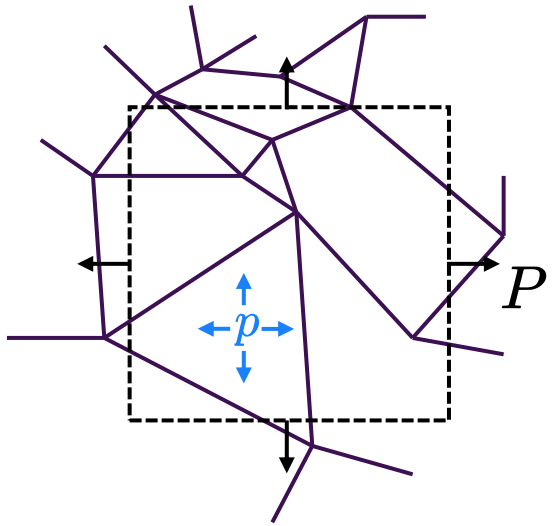
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In the fluid

$$\boldsymbol{\sigma}^{(f)} = -p\,\mathbf{I} + \text{viscous}$$

A scaling argument (slow rate-of-strain) allows us to neglect viscous stresses (but only in the stress tensor)

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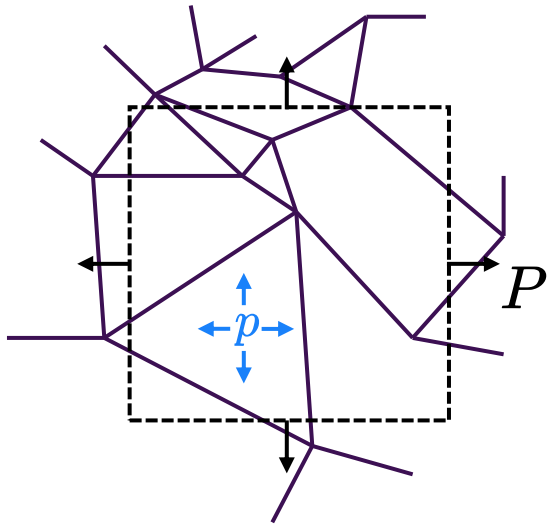
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In the bulk

Collect fluid pressure, elastic stress, general osmotic effects. Isotropically,

$$\operatorname{tr}\boldsymbol{\sigma} = -3P = -3p + \operatorname{tr}\boldsymbol{\sigma}^{(e)}$$

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Osmotic?

$$\Pi(\phi) = P - p \quad \Rightarrow \quad \boldsymbol{\sigma}^{(e)} = -\Pi(\phi)\mathbf{I} + \boldsymbol{\sigma}^{(\mathrm{dev})}$$

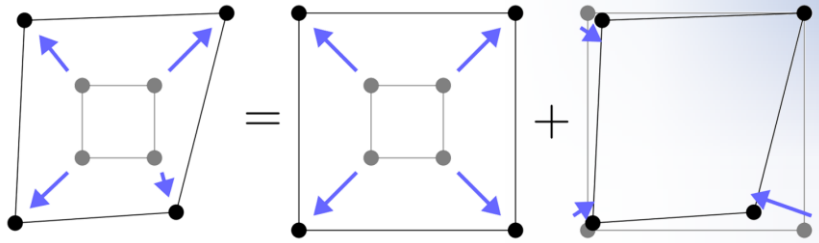
Isotropic elastic & hydrophilic effects captured by a generalised osmotic pressure

Peppin *et al.* (2005) *Phys. Fluids* ***17:***053301

Pressure and stress

JJW & Worster (2023) *J. Fluid Mech.* ***960:****A37*

This “generalised osmotic pressure” captures everything gel-like about the material



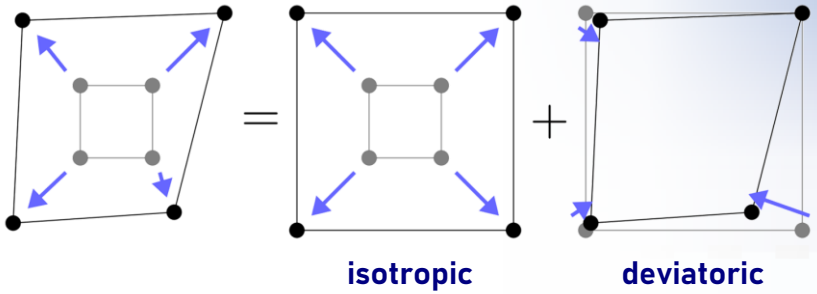
“measure of how and why a gel wants to take on or expel water”

- Needs to be zero at the uniform state
- There’s a maximum “drying state” where it will grow to be very large
- For small deformation, looks somewhat linear

Pressure and stress

JJW & Worster (2023) *J. Fluid Mech.* ***960:***A37

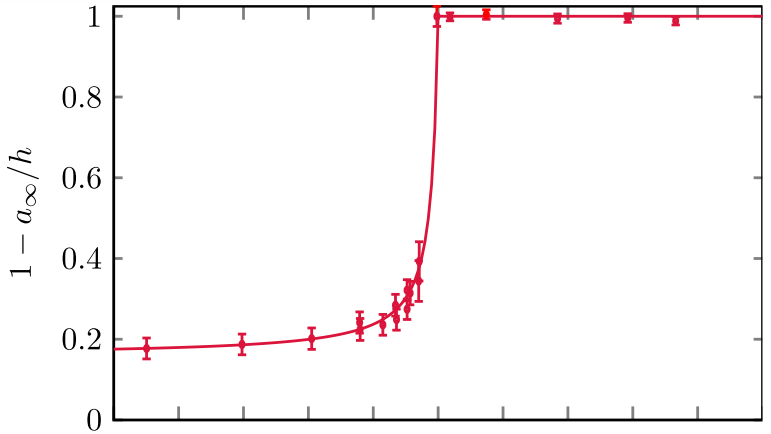
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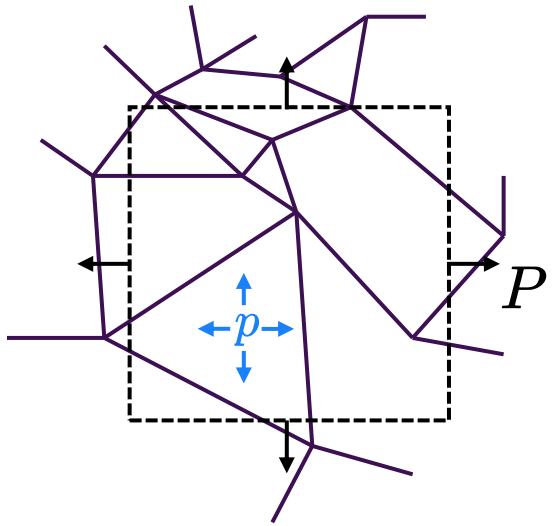
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data from **Feng *et al.*** J. Mech. Phys. Solids **201:**106166 (2025)



$$\Pi = \Pi_0 \frac{\phi/\phi_0 - 1}{1 - \phi/\phi_C}$$

Deviatoric stresses and strains



Making these assumptions,

$$\boldsymbol{\sigma} = -\left[p + \Pi(\phi)\right]\mathbf{I} + \boldsymbol{\sigma}^{(\mathrm{dev})}$$

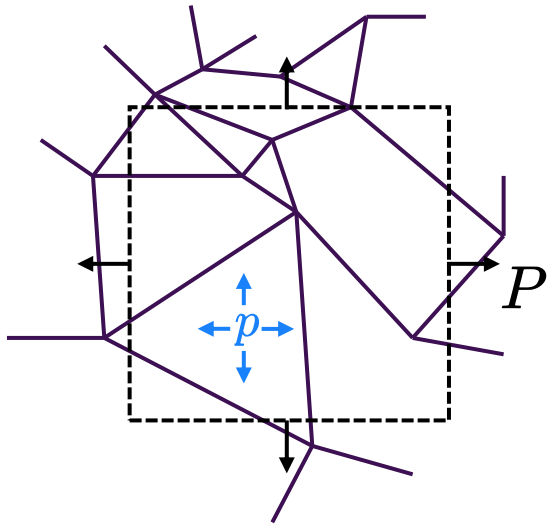
where $\boldsymbol{\sigma}^{(\mathrm{dev})} = f(\boldsymbol{\epsilon})$ - deviatoric stresses depend only on deviatoric strains. Making the simplest, linear elastic assumption,

$$\boldsymbol{\sigma} = -\left[p + \Pi(\phi)\right]\mathbf{I} + 2\mu_s(\phi)\boldsymbol{\epsilon}$$

shear modulus depends on swelling state (dry gels are stiffer)

effect of isotropic strains dealt with in *Π*

Deviatoric stresses and strains



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This is not linear elasticity – can capture nonlinearities in swelling state, which is important

This is a fluids seminar.

JJW & Worster (2023) *J. Fluid Mech.* ***960:***A37

…so we need to model flow through a gel – for which we use Darcy's law:

$$\mathbf{u} = -\frac{k(\phi)}{\mu_l}\boldsymbol{\nabla} p$$

$\mathbf{u}$: relative fluid flux

$\boldsymbol{\nabla} p$: gradients in pore pressure drive flow

Permeability

Clearly depends on solid fraction. How?

- **Kozeny-Carman**
  $\frac{(1-\phi)^3}{\phi^2} \approx \phi^{-2}$ idealised spherical particles; not very gel-like
- **Etzold *et al.*** (2021) *JFM* **925:**A8
  $\phi^{-2/3}$ small deswelling/swelling; *8*x volume shrinkage for *4*x permeability reduction!?
- **Feng *et al.*** (2026) *arXiv:2609.29904*
  $\phi^{-4.6}$

+ various pore-closure models…

Conservation of polymer + conservation of water gives

$$\frac{\partial \phi}{\partial t} + \mathbf{q}\cdot\boldsymbol{\nabla}\phi = \boldsymbol{\nabla}\cdot(\phi\mathbf{u})$$

$\mathbf{q}$: phase-averaged (solid+fluid) flux

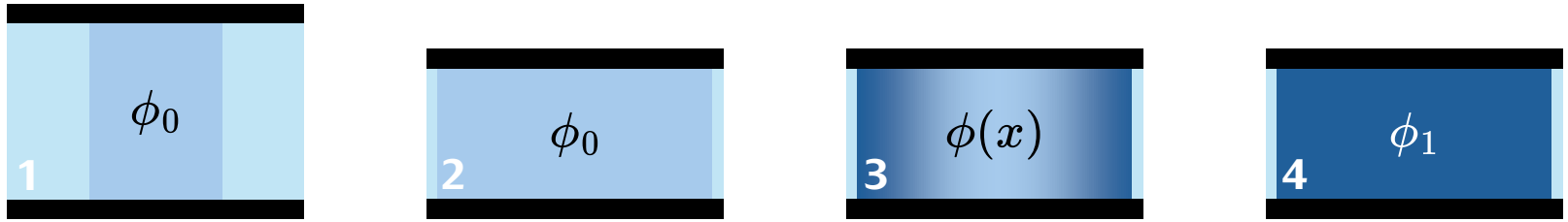
Governing parameters

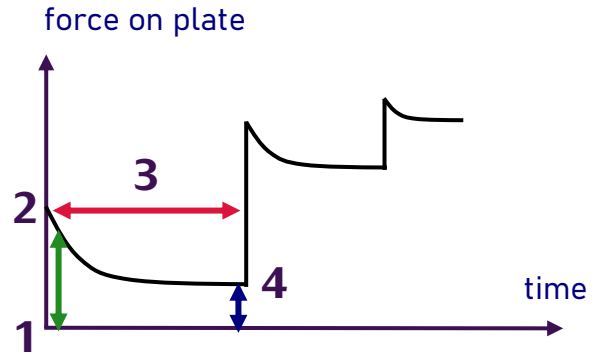
JJW & Worster (2023) *J. Fluid Mech.* ***960:****A37*

Shear modulus characterises the stiffness of a hydrogel and describes the initial elastic response before water diffuses through the structure

Osmotic pressure characterises the affinity for water ('desire' to swell or deswell)

Permeability describes the resistance to viscous flow through the pore scaffold





Governing equations

We look for steady state compression profiles so eliminate solid velocities and time derivatives

$$\frac{\partial \phi}{\partial t} + \mathbf{q} \cdot \boldsymbol{\nabla} \phi = \boldsymbol{\nabla} \cdot (\phi \mathbf{u})$$

$$\boldsymbol{\sigma} = -\left[p + \Pi(\phi)\right] \mathbf{I} + 2\mu_s(\phi)\boldsymbol{\epsilon}$$

$$\mathbf{u} = -\frac{k(\phi)}{\mu_l} \boldsymbol{\nabla} p$$

$$\boldsymbol{\nabla} \cdot \boldsymbol{\sigma} = \mathbf{0}$$

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Top surface

Match stress and continuity of pore pressure; remain fully swollen.

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Match pore pressure but stress not continuous.

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$$\frac{\partial}{\partial z}\left[\Phi^{-\alpha}\left(\frac{\partial \Pi}{\partial \Phi} + \frac{4\mathcal{M}}{3}\Phi^{-2/3}\right)\frac{\partial \Phi}{\partial z}\right] = 0$$

$$\Pi(\Phi_0) + 4\mathcal{M}(\Phi_0^{1/3} - 1) = \mathcal{P} \qquad \Phi_1 = 1$$

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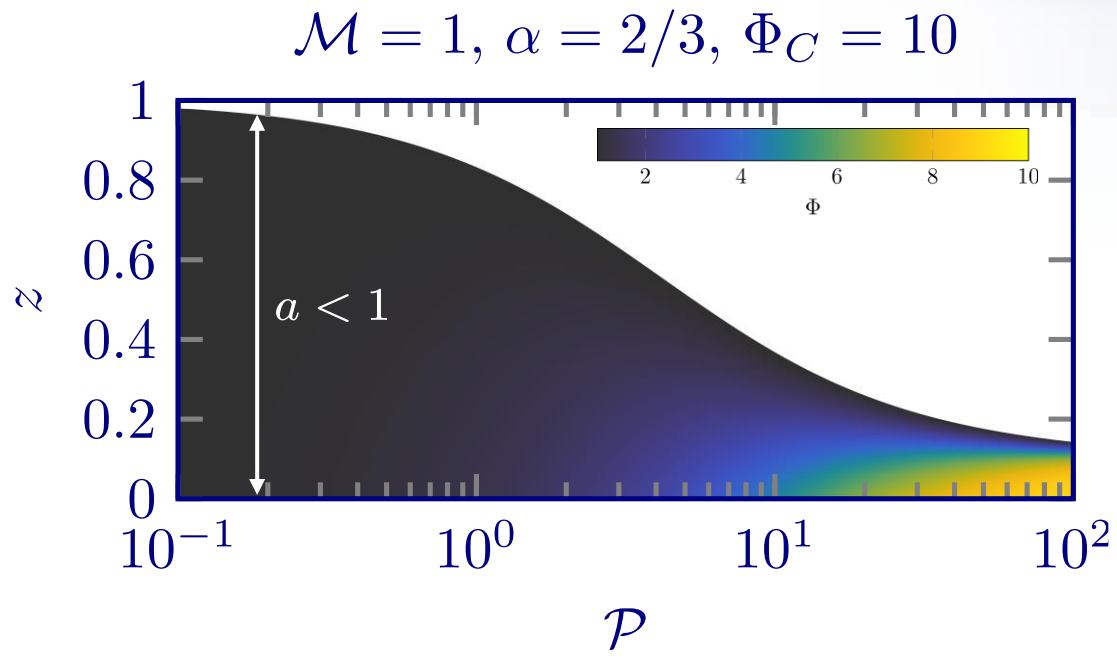
$\Phi = \phi/\phi_0$; scaled with Π_0; μ_s/Π_0; $\tilde{z}/a_0$

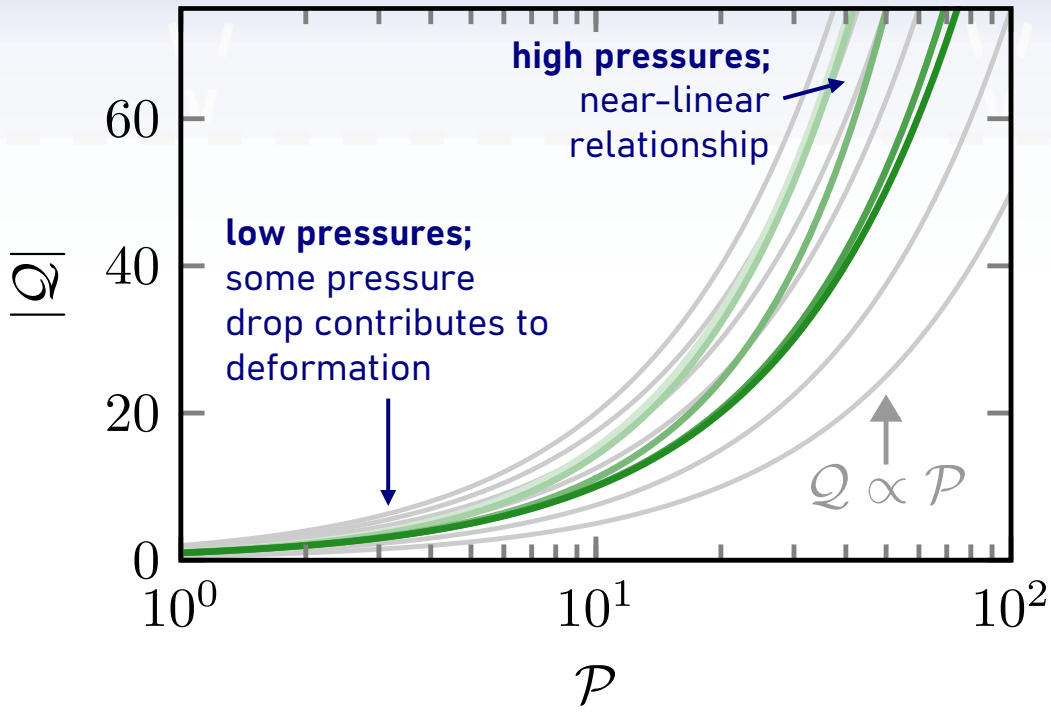
$$\Pi(\Phi_0) + 4\mathcal{M}(\Phi_0^{1/3} - 1) = \mathcal{P} \qquad \Phi_1 = 1$$

$\mathcal{P} = \Delta\tilde{p}/\Pi_0$

Compression under flow

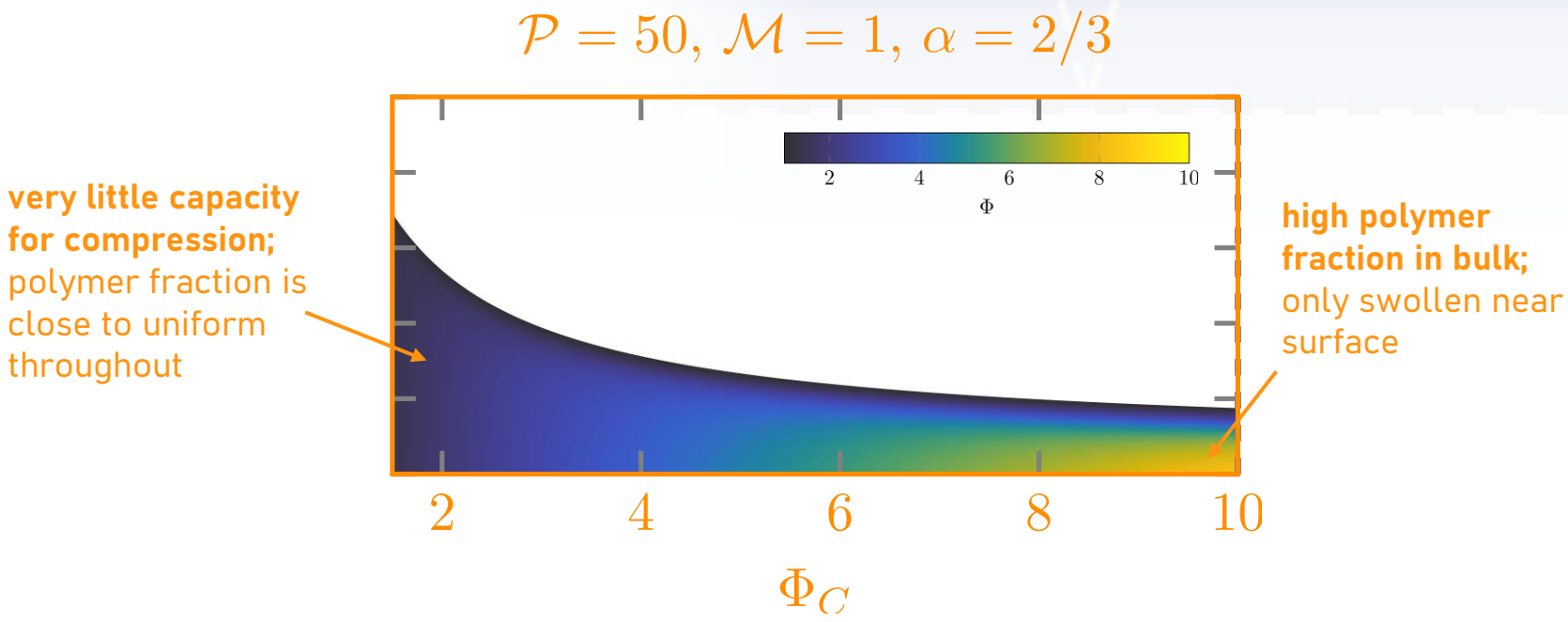
In steady state, the top remains swollen, whilst the base compacts more for a larger *P*





Compression under flow

There is more compaction if the maximum deswollen state is drier

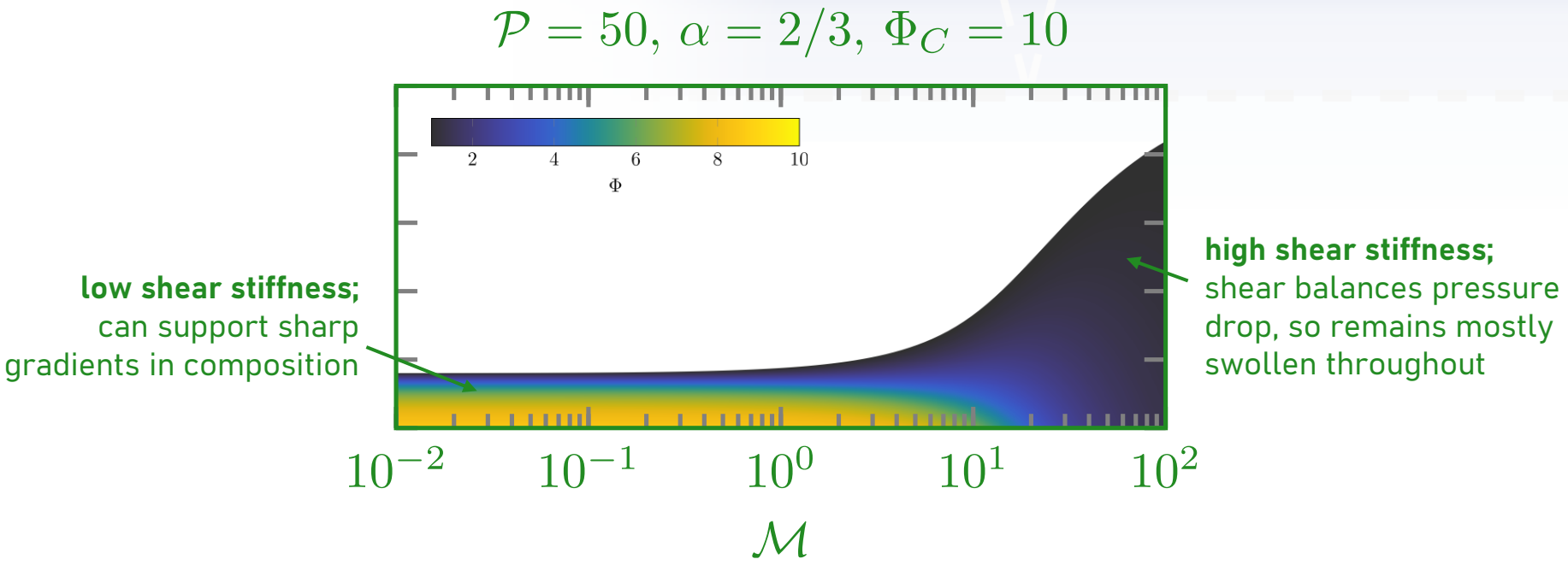


very little capacity for compression; polymer fraction is close to uniform throughout

high polymer fraction in bulk; only swollen near surface

Compression under flow

In steady state, the top remains swollen, whilst the base compacts more for a larger *P*

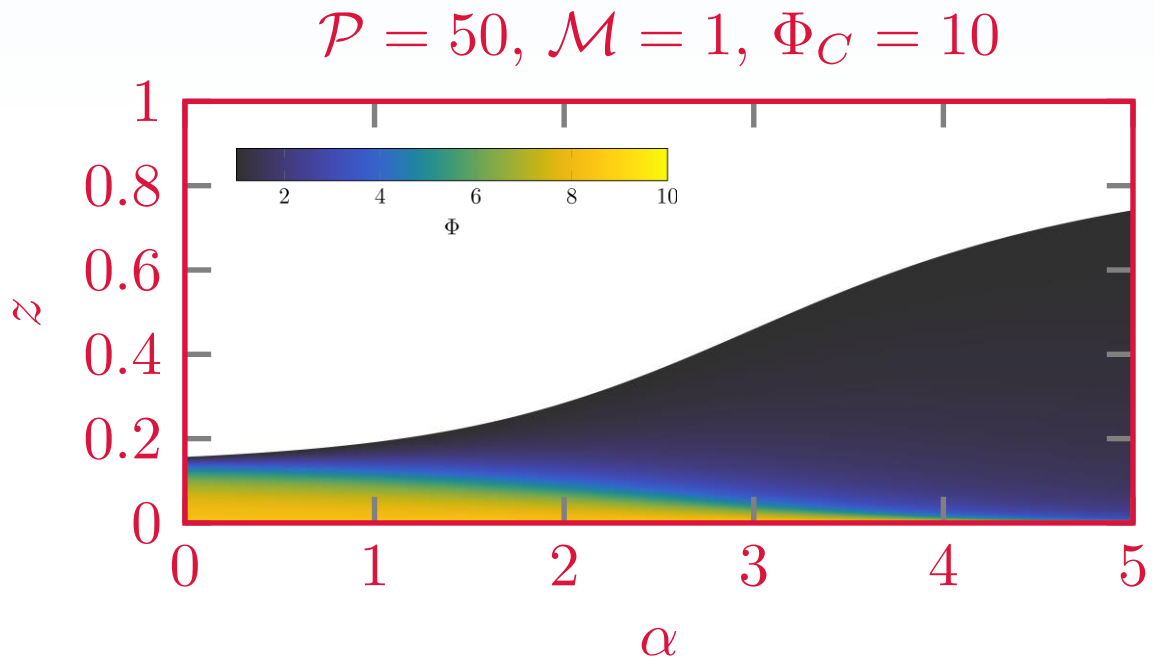


What about α?

Increasing the dependence of permeability on polymer fraction actually leads to less compaction. To see why,

$\Pi(\Phi_0) + 4\mathcal{M}(\Phi_0^{1/3} - 1) = \mathcal{P}$ (independent of α)

so the base is a much greater hydraulic bottleneck when α is increased



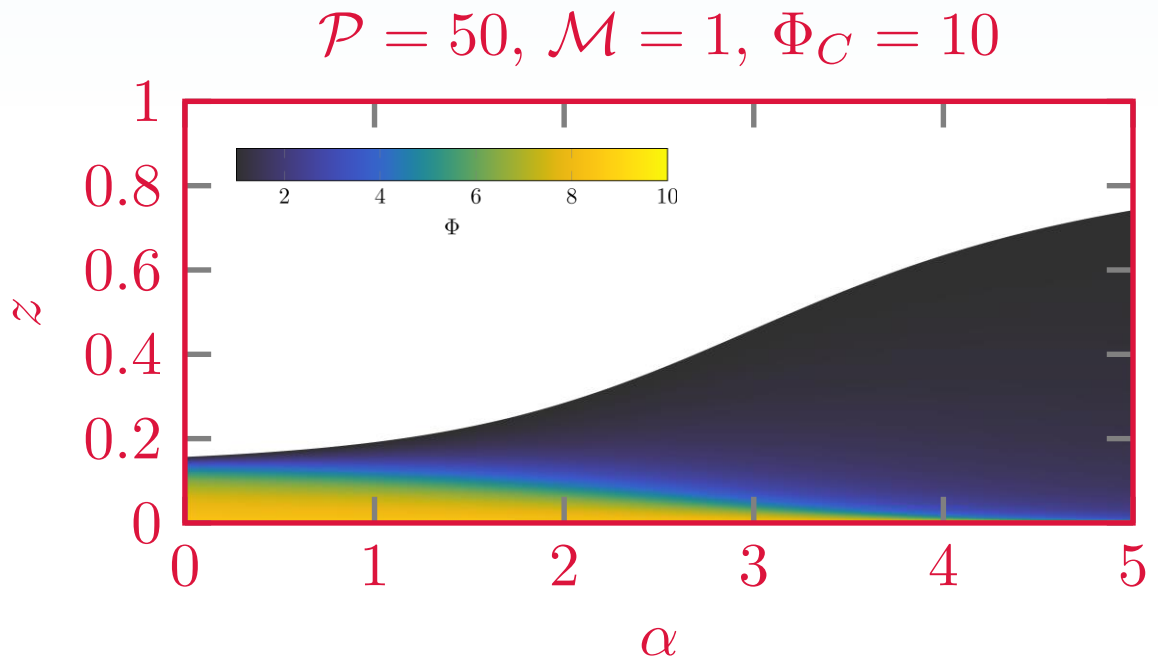
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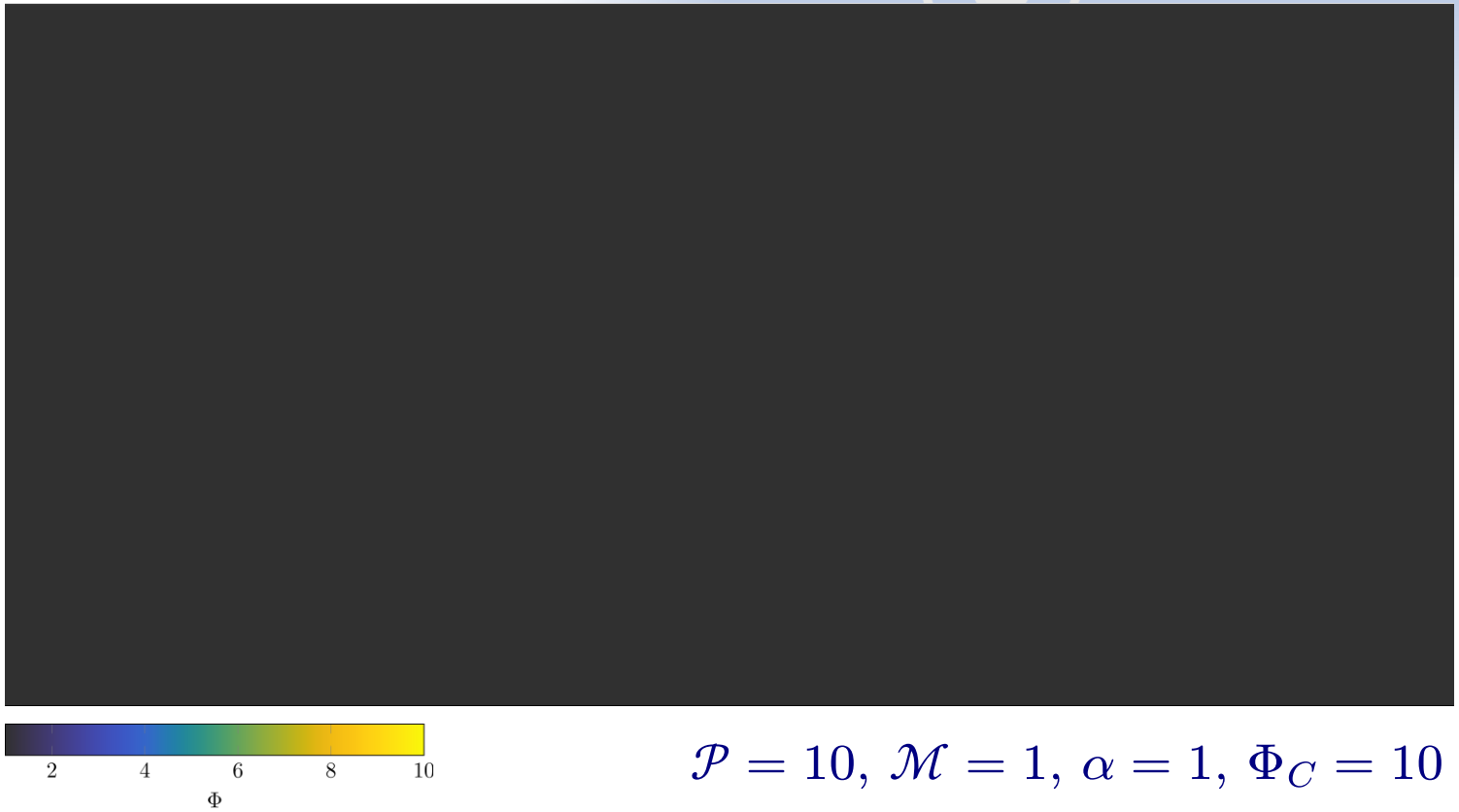
$\Pi(\Phi_0) + 4\mathcal{M}(\Phi_0^{1/3} - 1) = \mathcal{P}$ (independent of α)

so the base is a much greater hydraulic bottleneck when α is increased

the compacted layer 'shields' the rest of the gel from compaction, taking up much of the pressure drop



Do we ever see these profiles?



$\mathcal{P} = 10,\ \mathcal{M} = 1,\ \alpha = 1,\ \Phi_C = 10$

Do we ever see these profiles?

Unless you're careful, *granular* porous media will always channel when a pressure-driven flow is forced through

reddit.com/r/espresso

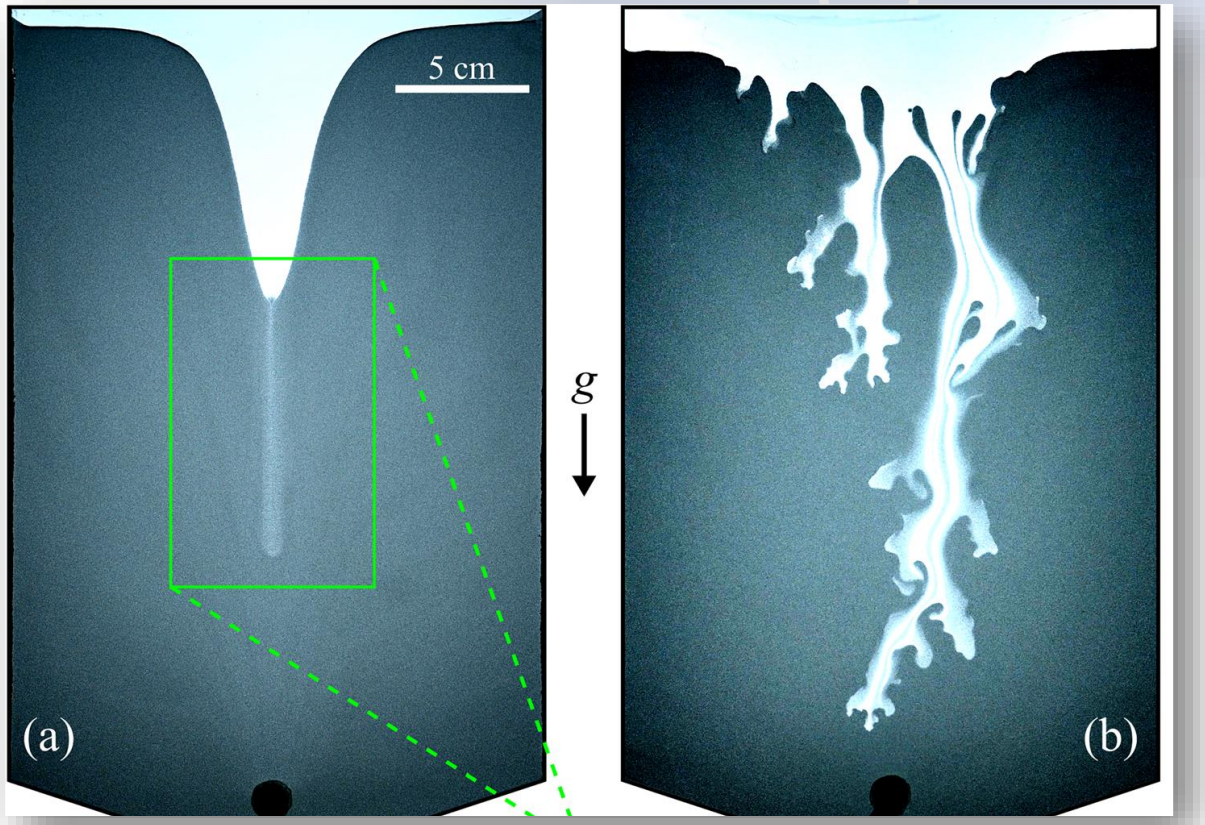




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Morgan et al. – *Nat. Comms. Phys.* ***8*** *(2025)*



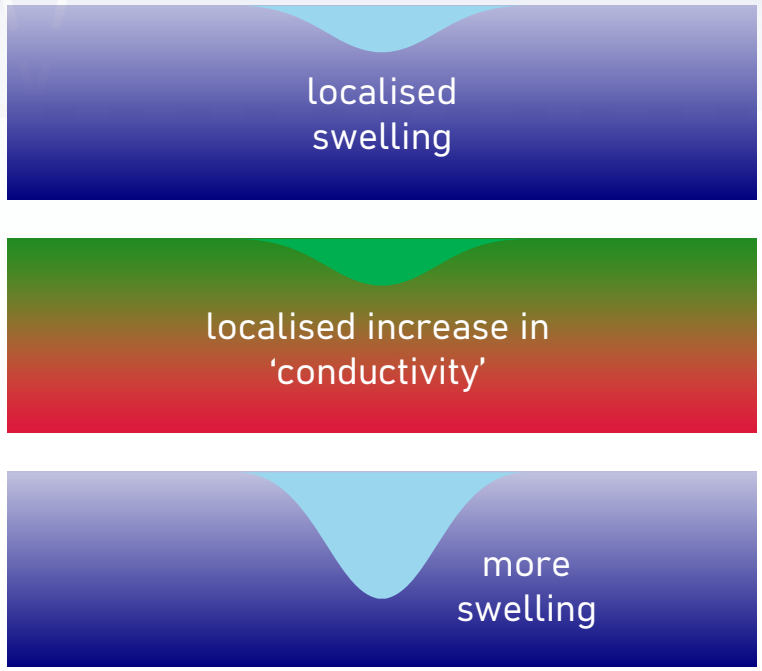
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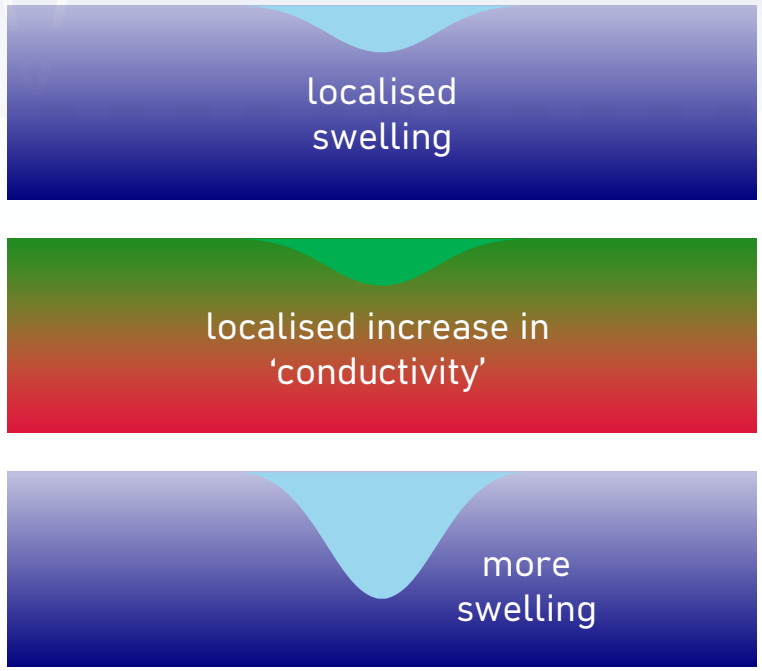
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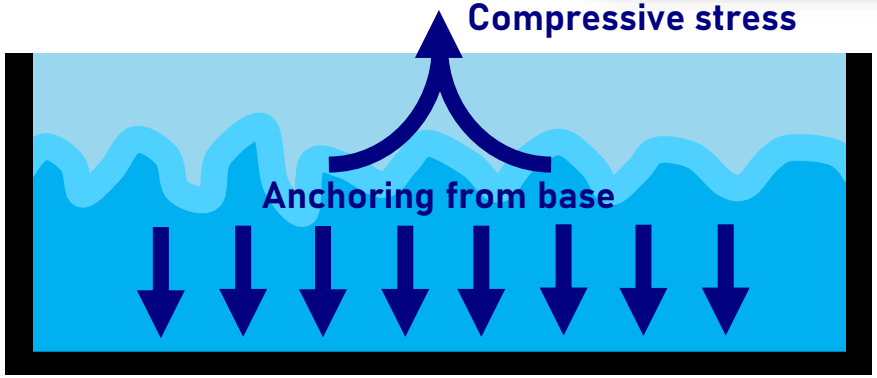


but the dynamics in cross-linked media are diffusive!

Instabilities in gels



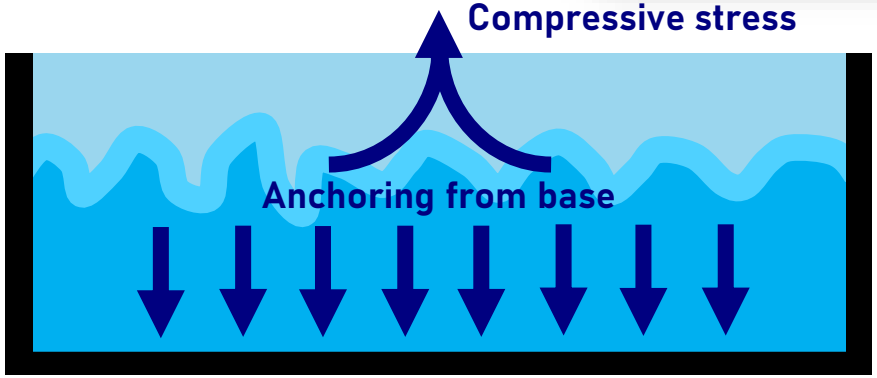




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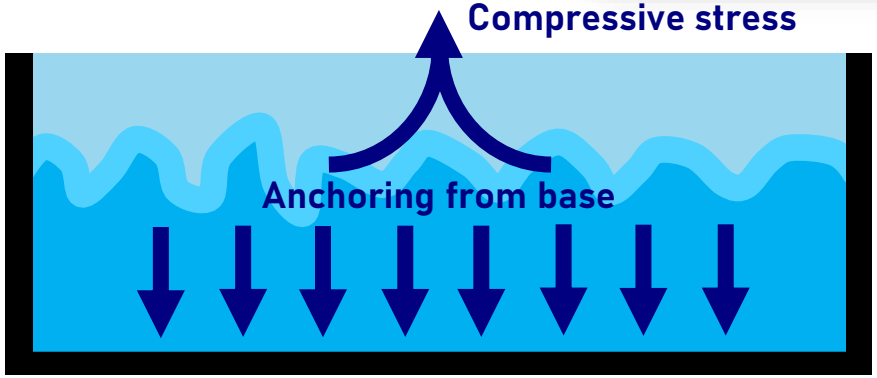


Elastic instability
Differential swelling gives regions of compressive stress that relieve through buckling (a bit like a beam)

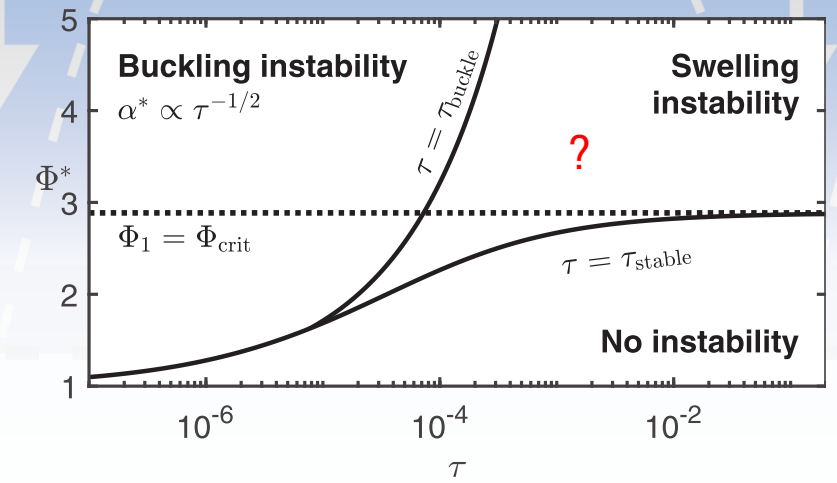
Instabilities in gels







Elastic instability
Differential swelling gives regions of compressive stress that relieve through buckling (a bit like a beam)



Swelling instability
Relief of deviatoric stresses drives preferential swelling in some regions that creates wrinkles

JJW & Worster, *Phys. Rev. E* **109**:044602 (2024)

Probing the stability

The stability problem is 2D – choose axes aligned with the “horizontal” deformation and let γ be the horizontal component of the wavevector

$$\boldsymbol{\xi} = \boldsymbol{\xi_0} + \epsilon\boldsymbol{\xi_1} = \eta_0(z)\hat{\mathbf{z}} + \varepsilon\left[\xi_{\parallel}(z) + \eta_1(z)\hat{\mathbf{z}}\right] e^{\mathrm{i}\mathbf{k}\cdot\mathbf{x}+st}$$

$$\boldsymbol{\xi} = \boldsymbol{\xi_0} + \epsilon\boldsymbol{\xi_1} = \eta_0(z)\hat{\mathbf{z}} + \varepsilon\left[\xi_{\parallel}(z)\hat{\mathbf{x}} + \eta_1(z)\hat{\mathbf{z}}\right] e^{\mathrm{i}\gamma x+st}$$

positive *s* gives instability;
γ gives the (reciprocal) horizontal lengthscale

Displacing elements from their steady state positions creates regions of swelling and deswelling

$$\Phi = \Phi_0(z) + \varepsilon\Phi_1(z)e^{\mathrm{i}\gamma x+st}$$

$$\mathrm{i}\gamma\xi_{\parallel}(z) + \eta_1'(z) = -\Phi_0^{-2/3}\Phi_1 = -\Psi(z)$$

$$\nabla\cdot\boldsymbol{\xi} = 3\left(1-\Phi^{1/3}\right)$$

Probing the stability

Elastic problem *2 equations from horizontal and vertical stress balance* $\nabla \cdot \sigma = 0$

$$\frac{\partial}{\partial z}\left[P_1 + \frac{\mathcal{M}}{3}\Psi(z)\right] = \mathcal{M}\left[\eta_1''(z) - \gamma^2\eta_1(z)\right] \qquad \mathrm{i}\gamma\left[P_1 + \frac{\mathcal{M}}{3}\Psi(z)\right] = \mathcal{M}\left[\xi_\parallel''(z) - \gamma^2\xi_\parallel(z)\right]$$

perturbation total pressure
pore and osmotic effects summed

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perturbation total pressure
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Swelling problem *1 equation*

$\frac{\partial \phi}{\partial t} + \mathbf{q}\cdot\nabla\phi = \nabla\cdot(\phi\mathbf{u})$

$$\mathcal{L} = \frac{\partial^2}{\partial z^2} - \gamma^2$$

$$\Phi_0^{-\alpha}\Phi_0' p_1' - \Phi_0^{-(\alpha-1)}\mathcal{L}[p_1] - \mathcal{Q}\alpha\Phi_0\frac{\partial}{\partial z}\left(\Phi_0^{-1/3}\Psi\right) = s\Phi_0^{2/3}\left(\Psi + \Phi_0^{-1}\Phi_0'\eta_1\right)$$

perturbation pore pressure

flux through base state

Probing the stability

Elastic problem *2 equations from horizontal and vertical stress balance* $\nabla \cdot \sigma = 0$

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perturbation pore pressure

flux through base state

$$\mathcal{L}[Z] = 0 \qquad \mathcal{M}\mathcal{L}[\eta_1] = Z' - \mathcal{M}\Psi' \qquad \mathcal{L}\left[\left(\frac{4\mathcal{M}}{3} + \left.\frac{\mathrm{d}\Pi}{\mathrm{d}\Phi}\right|_{\Phi_0}\Phi_0^{2/3}\right)\Psi\right] = s\Phi_0^{\alpha-1/3}\left(\Psi + \Phi_0^{-1}\Phi_0'\eta_1\right) + \mathcal{Q}\alpha\Phi_0^{\alpha}\frac{\partial}{\partial z}\left(\Phi_0^{-1/3}\Psi\right) -$$

$$Z = P_1 + \frac{4\mathcal{M}}{3}\Psi \qquad \Psi = \Phi_0^{-2/3}\Phi_1 \qquad \mathcal{Q} = \left(\left.\frac{\mathrm{d}\Pi}{\mathrm{d}\Phi}\right|_{\Phi=1} + \frac{4\mathcal{M}}{3}\right)\Phi_0'|_{z=a} \qquad \Phi_0^{-1}\Phi_0'\frac{\partial}{\partial z}\left[\left(\frac{4\mathcal{M}}{3} + \left.\frac{\mathrm{d}\Pi}{\mathrm{d}\Phi}\right|_{\Phi_0}\Phi_0^{2/3}\right)\Psi\right]$$

Probing the stability

Base *3 conditions*

- No displacement

$$\eta_1(0) = 0; \quad \Psi(0) + \eta_1'(0) = 0$$

- Continuity of pressure

$$Z(0) = \left[\Phi_0(0)^{2/3} + \frac{4\mathcal{M}}{3}\right]\Psi(0)$$

Free surface *3 conditions*

- Continuity of pressure

$$Z(a) = \left(1 + \frac{4\mathcal{M}}{3}\right)\Psi(a) + \mathcal{Q}\eta_1(a)$$

- Continuity of stress

$$Z'(a) + 2\mathcal{M}\gamma^2\eta_1(a) = 0; \quad Z(a) = 2\mathcal{M}\left[\eta_1'(a) + \Psi(a)\right]$$

$$\mathcal{L}[Z] = 0 \qquad \mathcal{M}\mathcal{L}[\eta_1] = Z' - \mathcal{M}\Psi' \qquad \mathcal{L}\left[\left(\frac{4\mathcal{M}}{3} + \left.\frac{\mathrm{d}\Pi}{\mathrm{d}\Phi}\right|_{\Phi_0}\Phi_0^{2/3}\right)\Psi\right] = s\Phi_0^{\alpha-1/3}\left(\Psi + \Phi_0^{-1}\Phi_0'\eta_1\right) + \mathcal{Q}\alpha\Phi_0^{\alpha}\frac{\partial}{\partial z}\left(\Phi_0^{-1/3}\Psi\right) -$$

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Probing the stability

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Normalisation *1 condition*

$$\int_0^a |Z|^2 + |\Psi|^2 + |\eta_1|^2 + \gamma^{-2}|\Psi + \eta_1'|^2 \,\mathrm{d}z = 1$$

$$\mathcal{L}[Z] = 0 \qquad \mathcal{M}\mathcal{L}[\eta_1] = Z' - \mathcal{M}\Psi' \qquad \mathcal{L}\left[\left(\frac{4\mathcal{M}}{3} + \left.\frac{\mathrm{d}\Pi}{\mathrm{d}\Phi}\right|_{\Phi_0}\Phi_0^{2/3}\right)\Psi\right] = s\Phi_0^{\alpha-1/3}\left(\Psi + \Phi_0^{-1}\Phi_0'\eta_1\right) + \mathcal{Q}\alpha\Phi_0^{\alpha}\frac{\partial}{\partial z}\left(\Phi_0^{-1/3}\Psi\right) -$$

$$Z = P_1 + \frac{4\mathcal{M}}{3}\Psi \qquad \Psi = \Phi_0^{-2/3}\Phi_1 \qquad \mathcal{Q} = \left(\left.\frac{\mathrm{d}\Pi}{\mathrm{d}\Phi}\right|_{\Phi=1} + \frac{4\mathcal{M}}{3}\right)\Phi_0'|_{z=a} \qquad \Phi_0^{-1}\Phi_0'\frac{\partial}{\partial z}\left[\left(\frac{4\mathcal{M}}{3} + \left.\frac{\mathrm{d}\Pi}{\mathrm{d}\Phi}\right|_{\Phi_0}\Phi_0^{2/3}\right)\Psi\right]$$

Horizontal banding instability

Solving this full problem is nasty, so let's make two simplifying assumptions:

- **Osmotic pressure is linear;** $\Pi = \Phi - 1$
  Under small deformation we don't envisage nonlinearity to be important, so qualitatively this doesn't change matters.

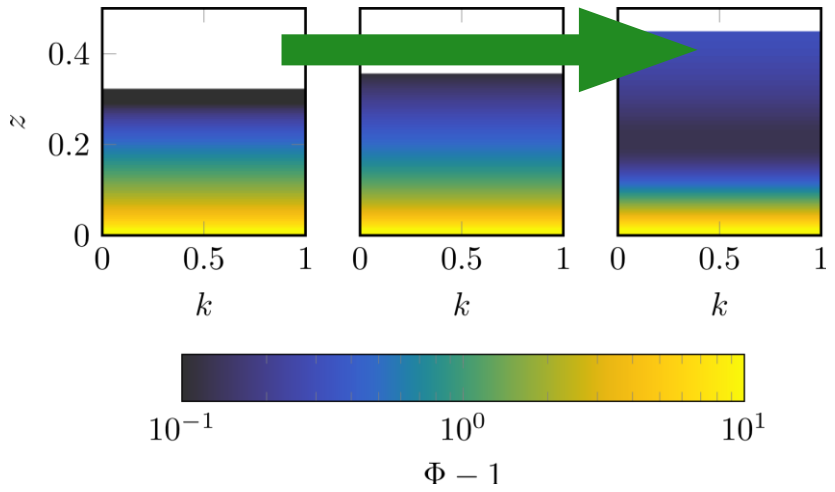
- **The deformation is horizontally uniform;** $\gamma = \xi_{/\!/} = 0$
  Before modelling "channels", consider only vertical displacement to get a handle on the mechanism for instability.

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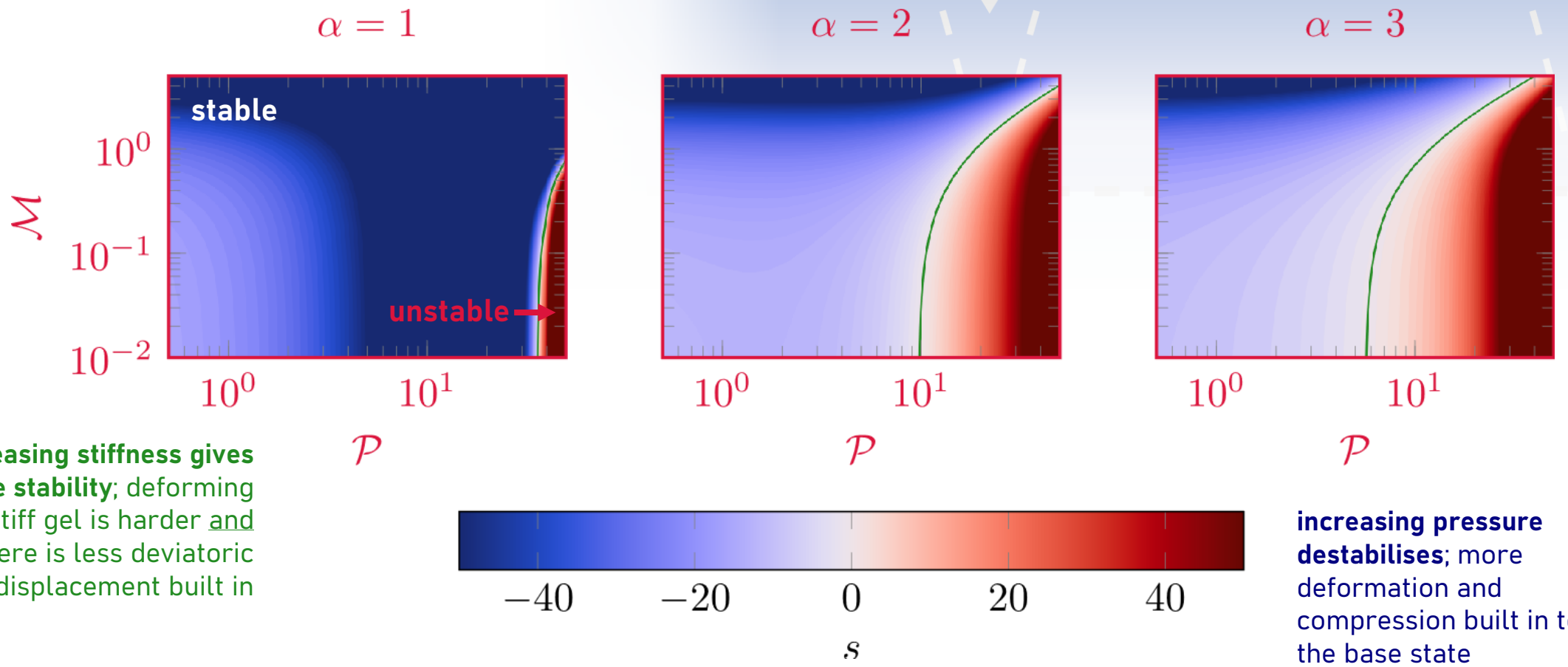
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Horizontal banding instability

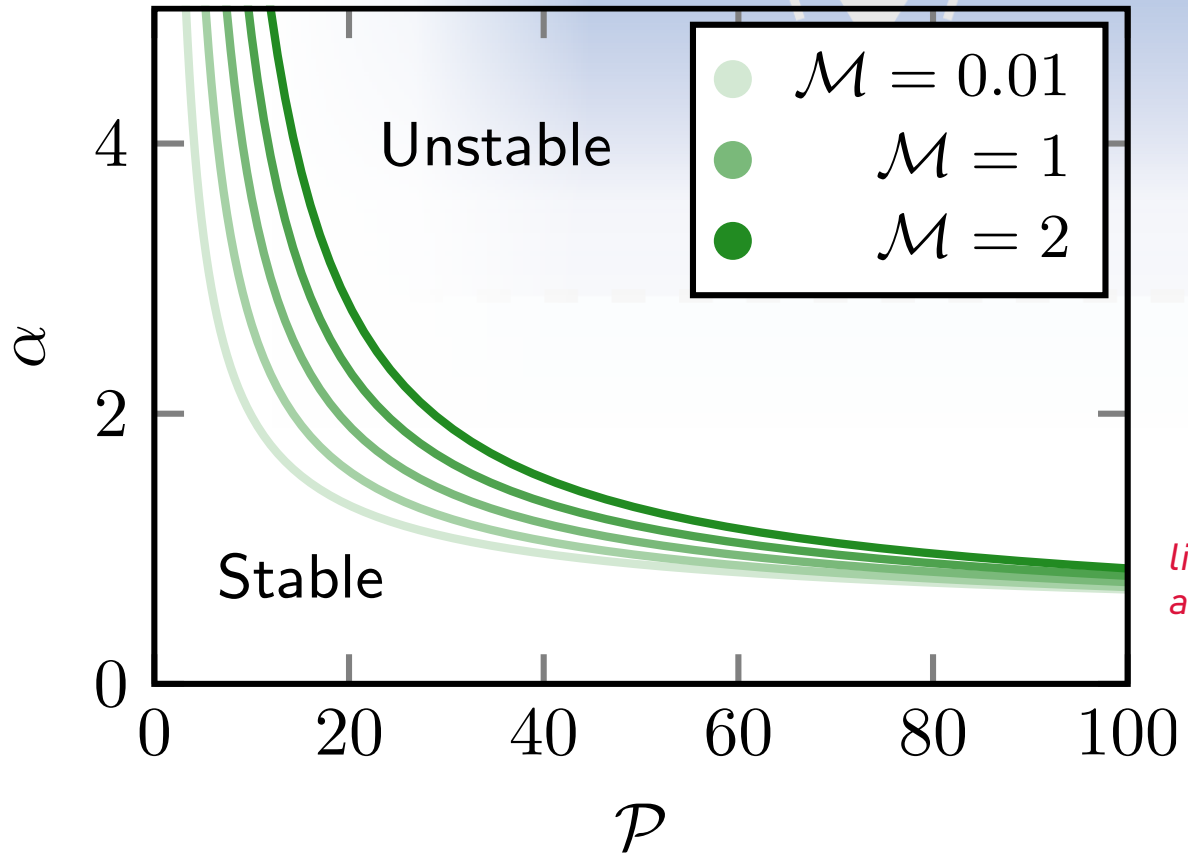
increasing permeability dependence on swelling gives less stability; the shielding effect is greater



increasing stiffness gives more stability; deforming a stiff gel is harder and there is less deviatoric displacement built in

increasing pressure destabilises; more deformation and compression built in to the base state

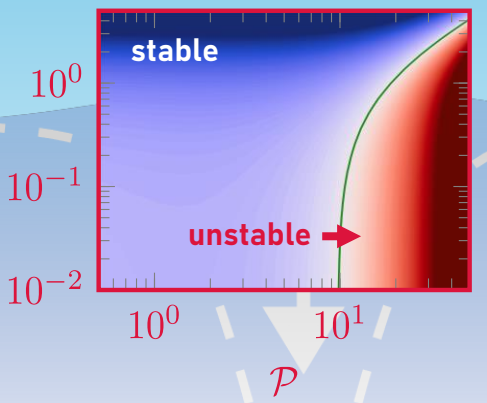
Horizontal banding instability



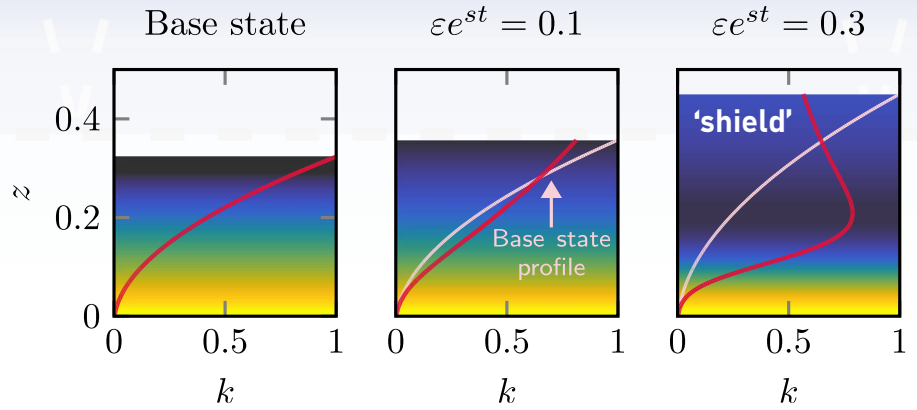
limit appears to be α=2/3 as M approaches zero

Horizontal banding instability

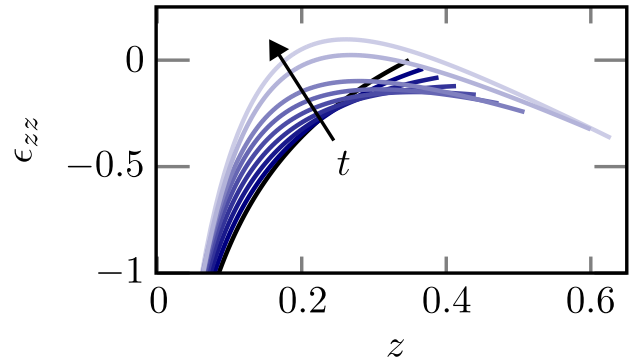
Playing around with these instabilities shows two key things are going on:



- **Nonzero *α* is necessary for shielding**
  This instability grows by deswelling some regions which ‘shield’ the rest from the pressure drop, allowing them to reswell

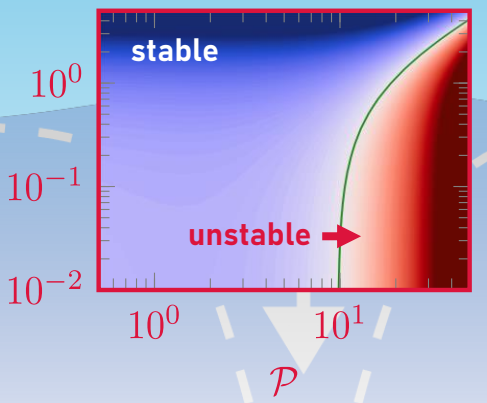


- **Banding relieves deviatoric stresses**
  But this doesn’t mean growth rates grow with stiffness because a stiff gel has less deviatoric strain baked into the base state



Horizontal banding instability

Can we justify this instability in terms of energy?

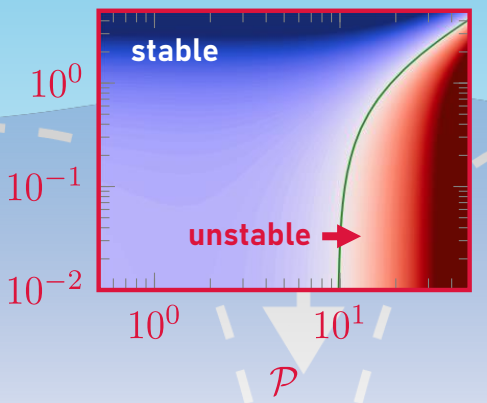


Wrinkling instabilities

- Inhomogeneous swelling introduces elastic stresses in the partly-swollen material
- The swollen surface is anchored to a drier base, and the total elastic energy stored in the gel is released by the formation of wrinkles/buckles/creases/folds…

Horizontal banding instability

Can we justify this instability in terms of energy?



Wrinkling instabilities

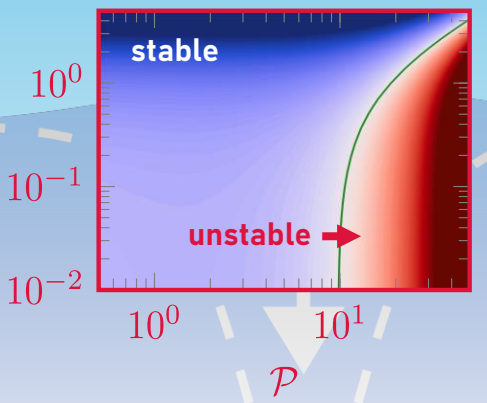
- Inhomogeneous swelling introduces elastic stresses in the partly-swollen material
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Our situation

- There are indeed elastic stresses in the compacted base state, and these are balanced by the work done driving the fluid flow (viscous resistance)
- This instability grows as a fluid-mechanical effect, not an elastic snap (though we've seen some deviatoric stresses may be relieved along the way)

Horizontal banding instability

Can we justify this instability in terms of energy?



Wrinkling instabilities

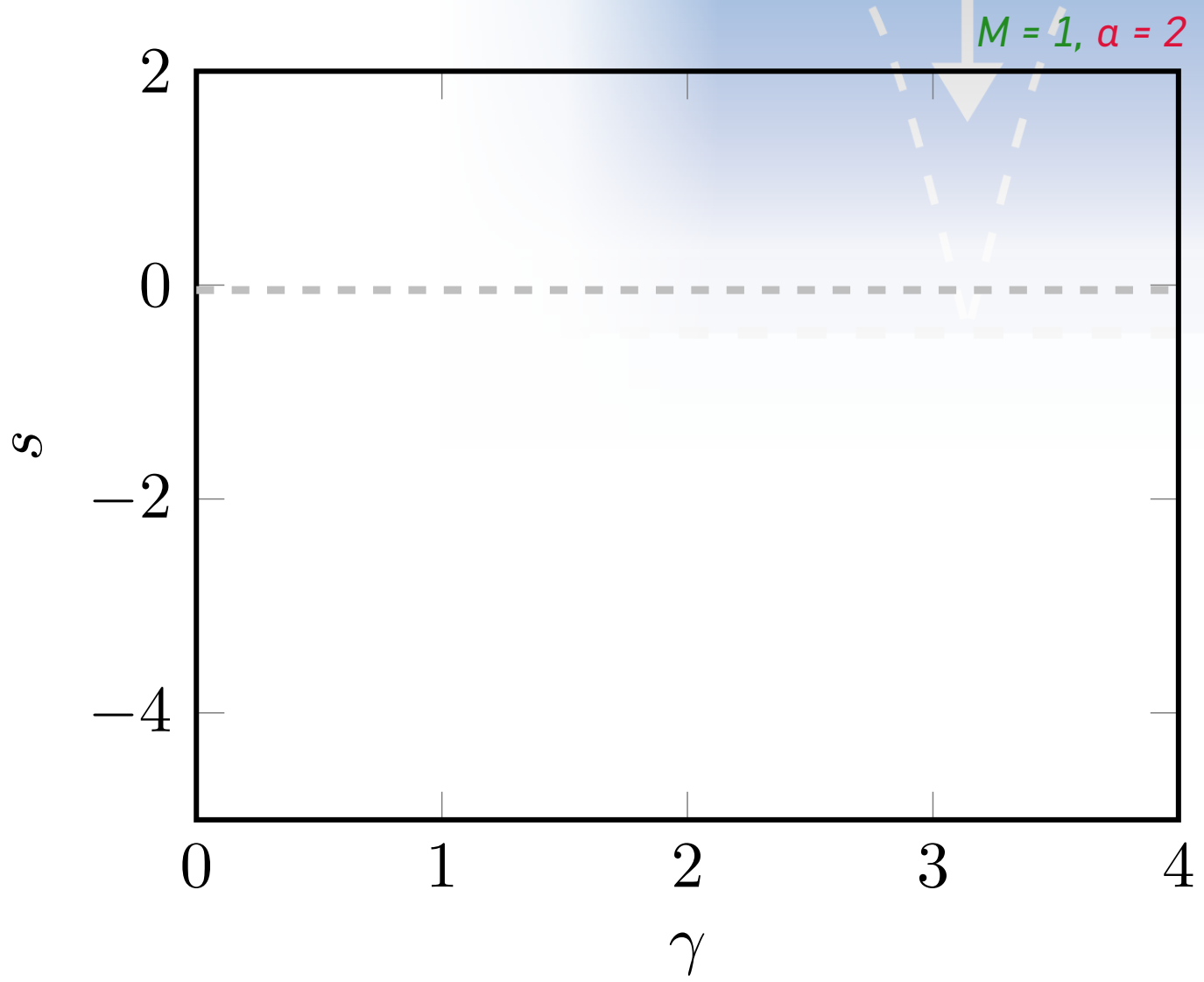
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- The swollen surface is anchored to a drier base, and the total elastic energy stored in the gel is released by the formation of wrinkles/buckles/creases/folds…

What resists instability?
The base is fixed and so there is forced anisotropy in any banding pattern formation…

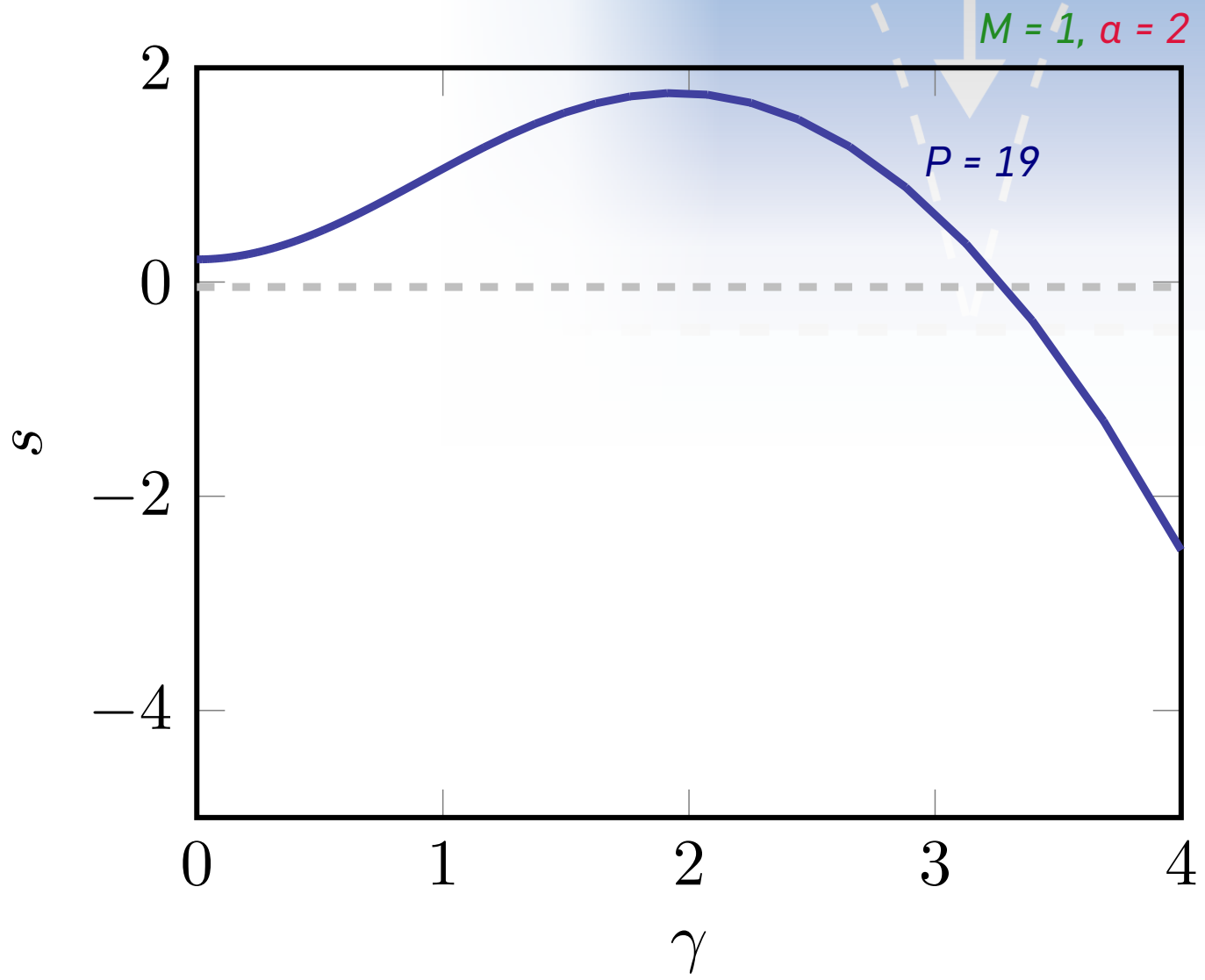
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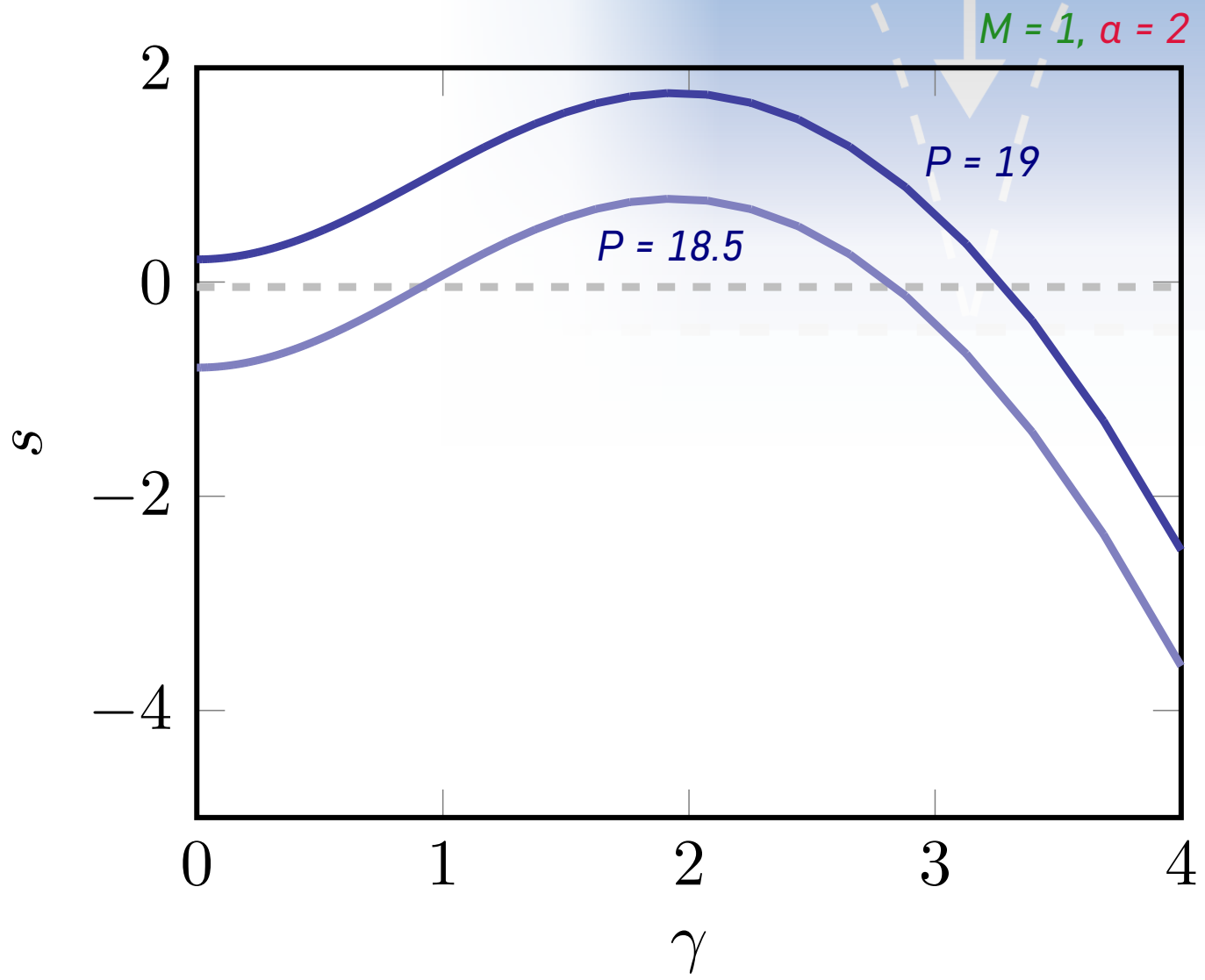
From banding to channelling



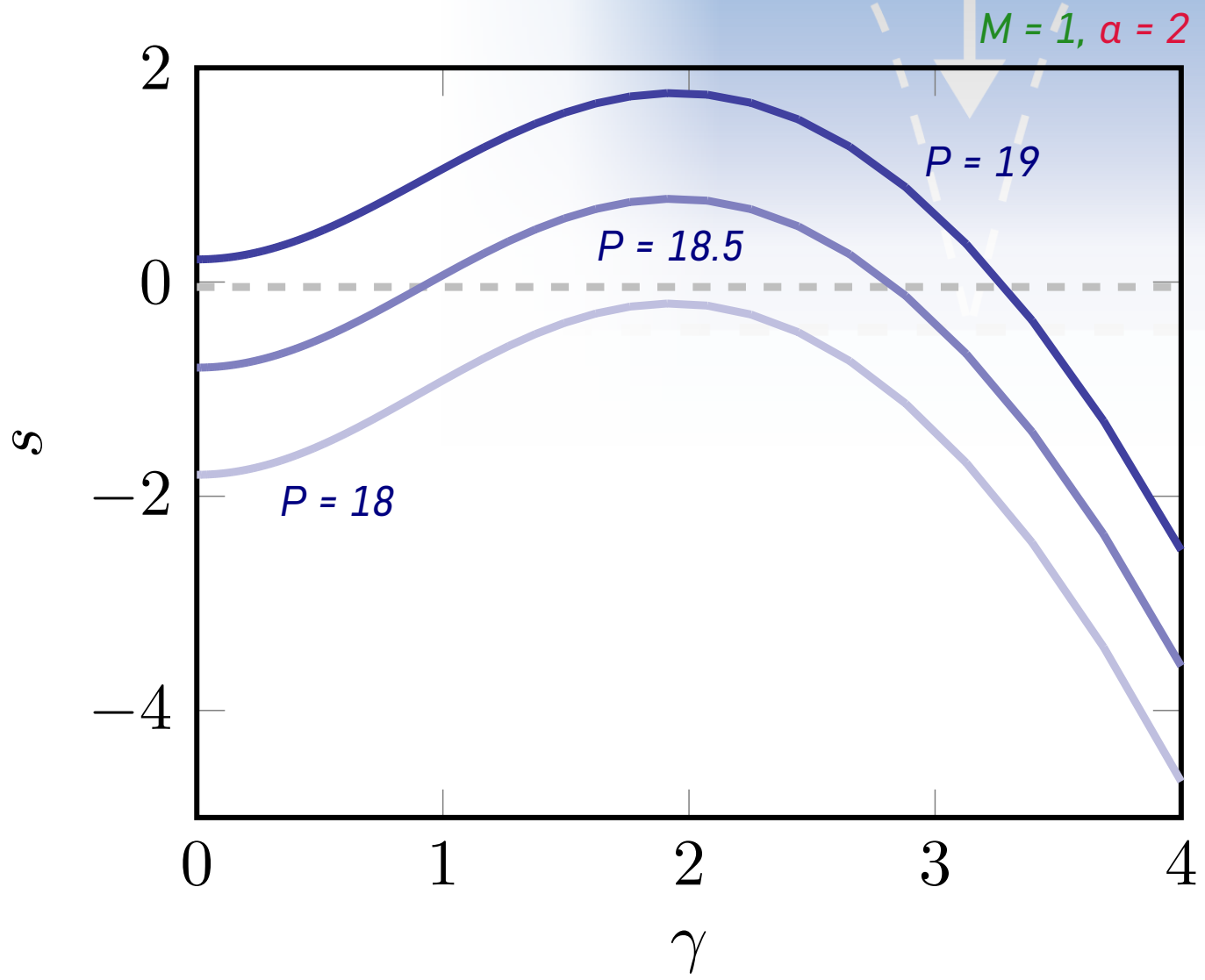
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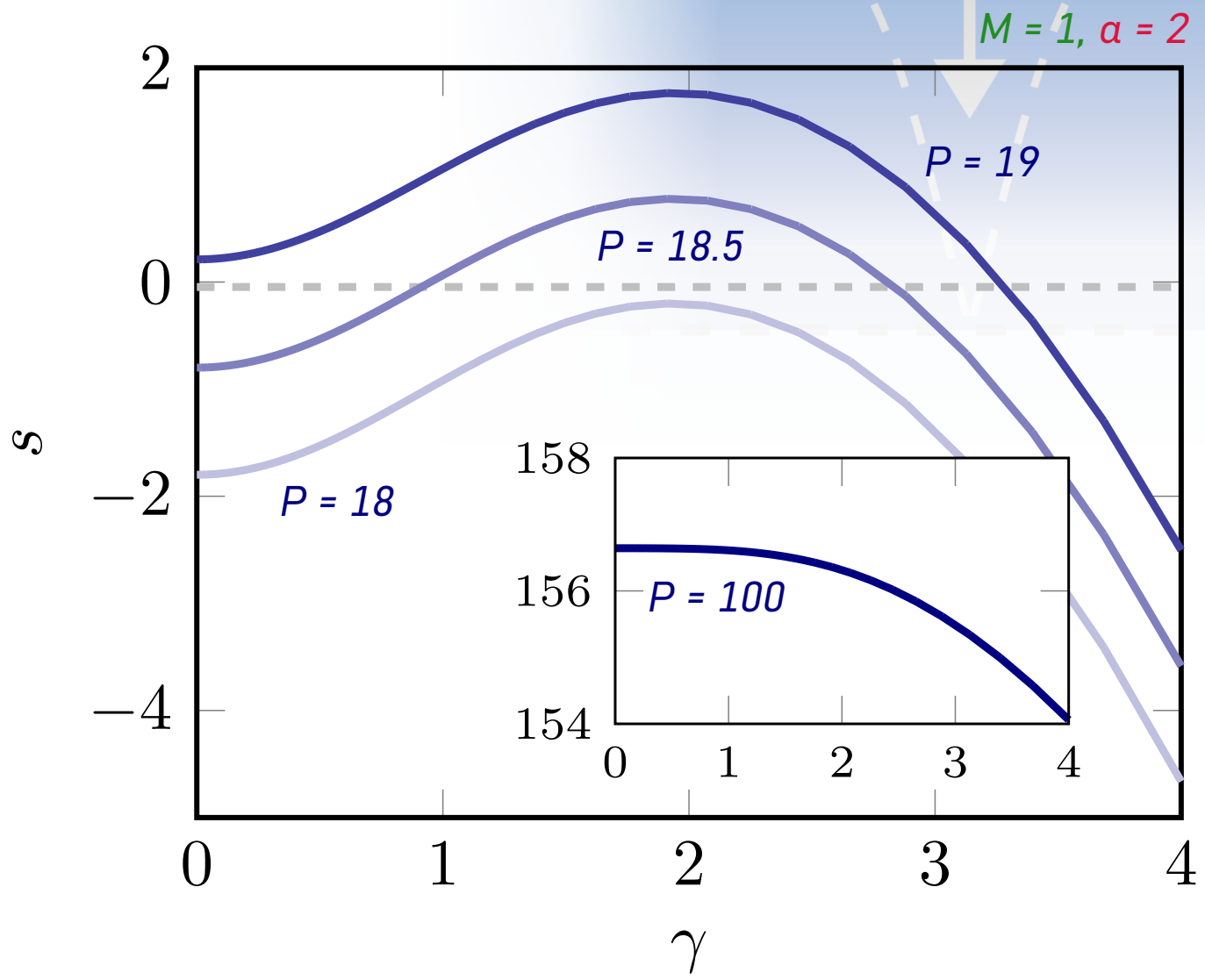
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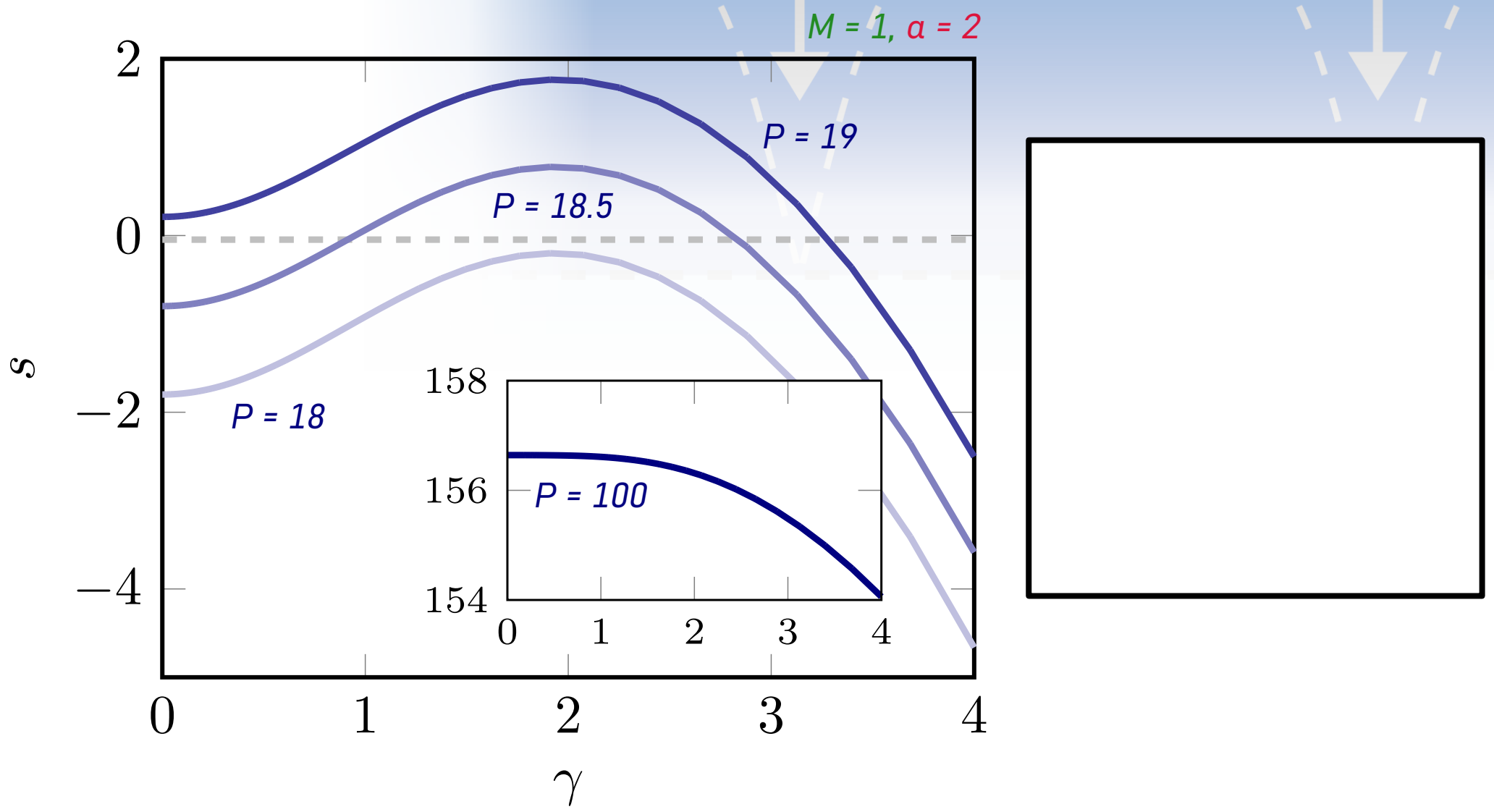
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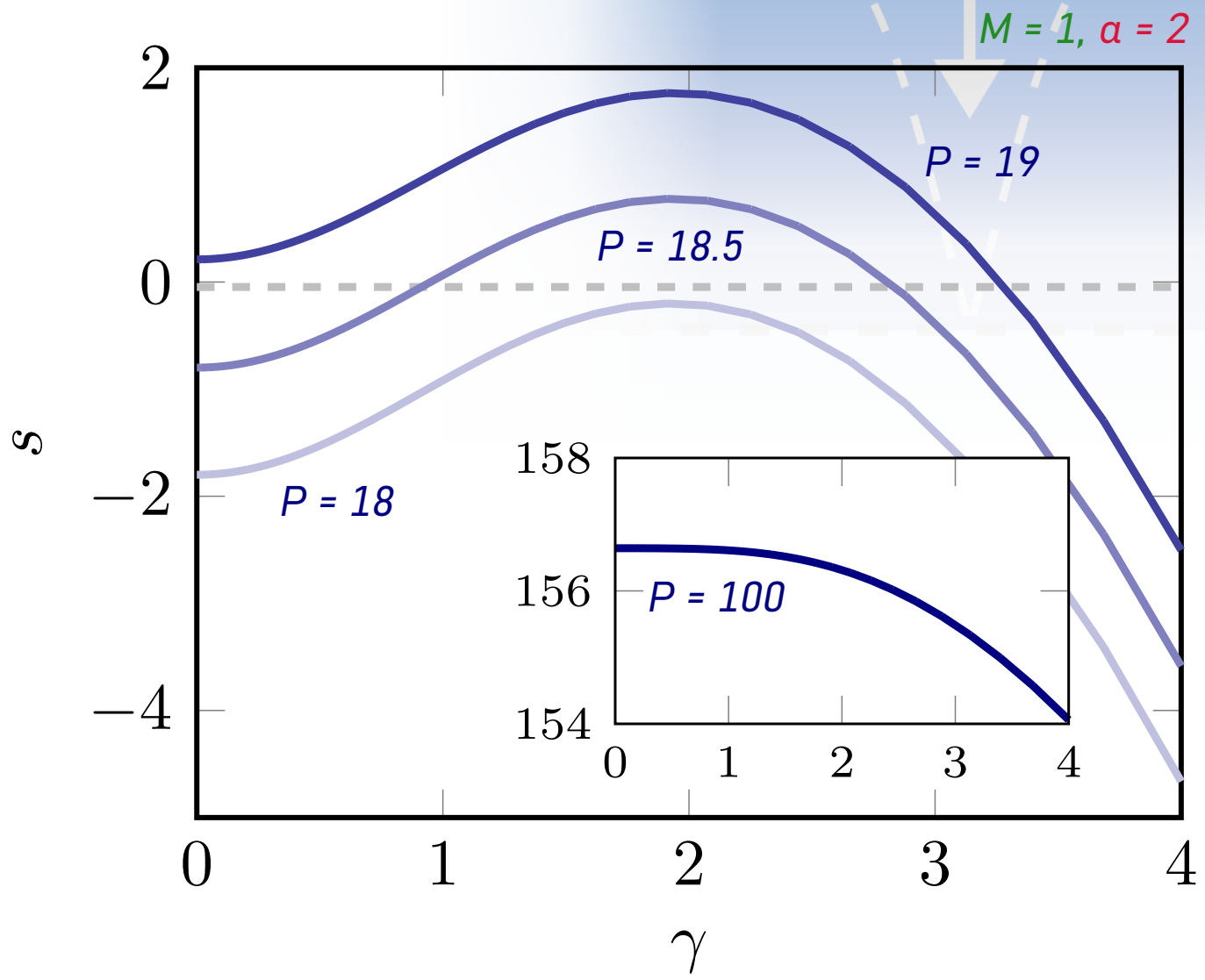
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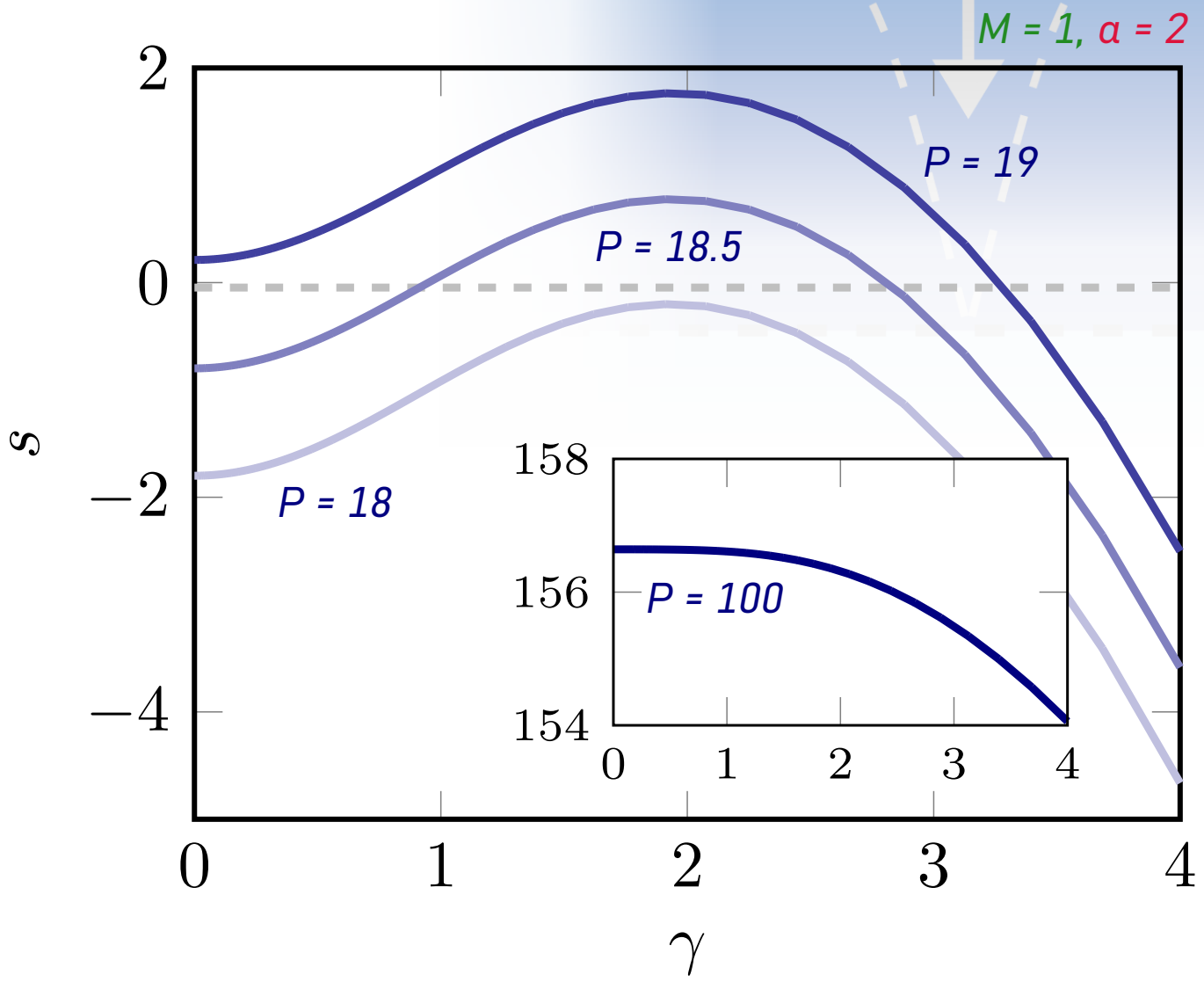


From banding to channelling



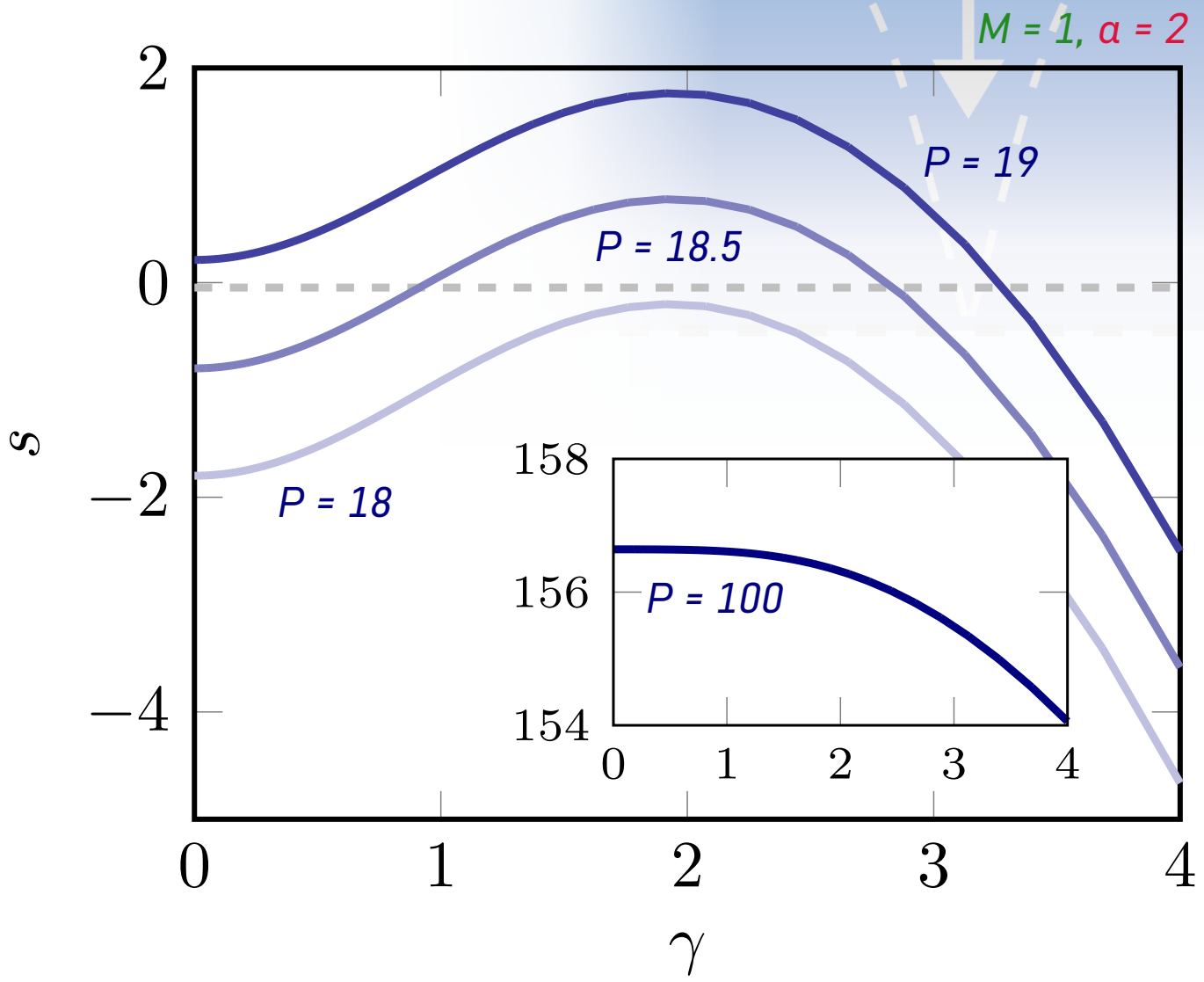
1. Neither banding nor channelling unstable

From banding to channelling



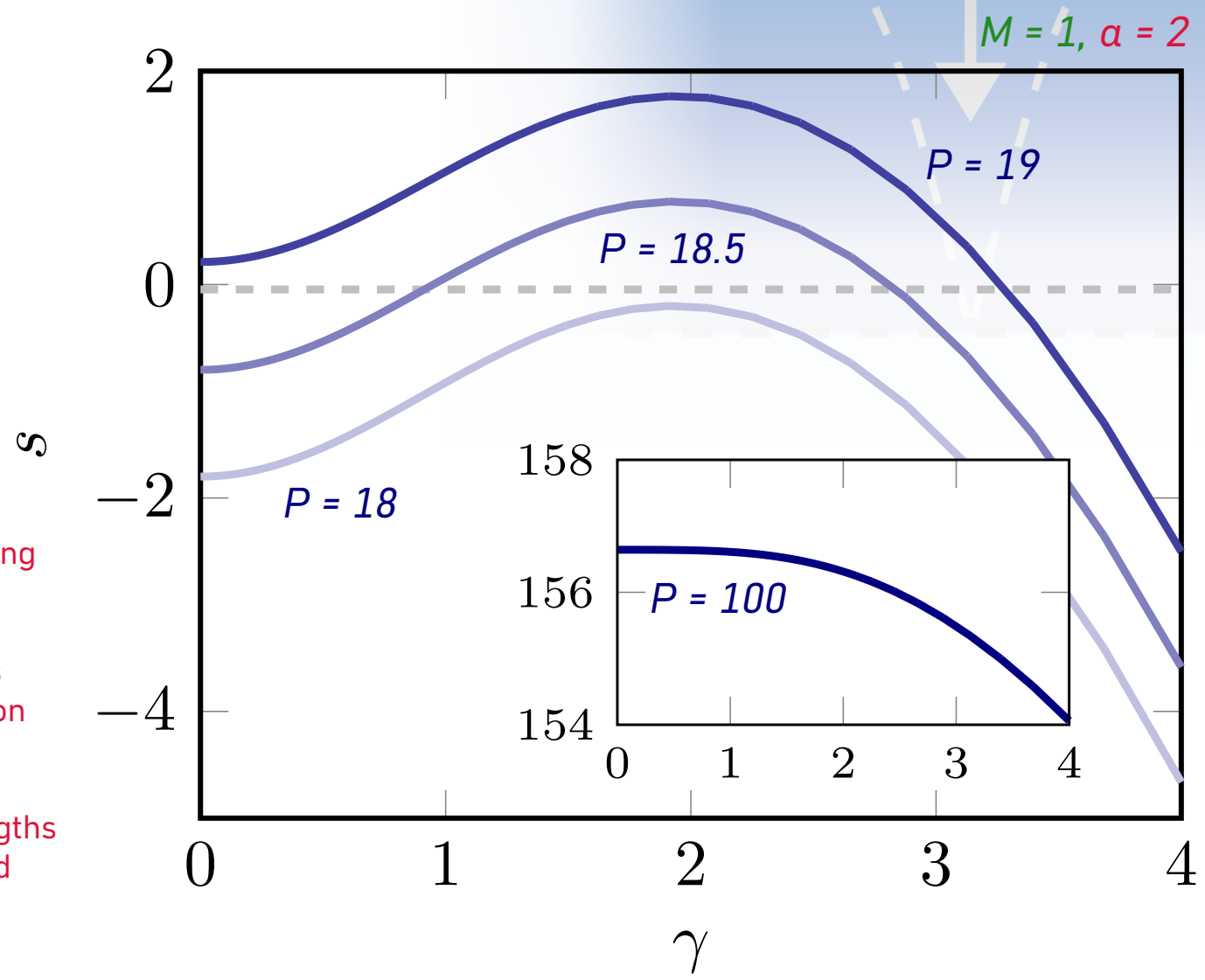
1. Neither banding nor channelling unstable
2. Banding not seen, but channelling unstable

From banding to channelling



1. Neither banding nor channelling unstable
2. Banding not seen, but channelling unstable
3. Both unstable:
   a) Channelling most unstable behaviour
   b) Banding most unstable behaviour

From banding to channelling



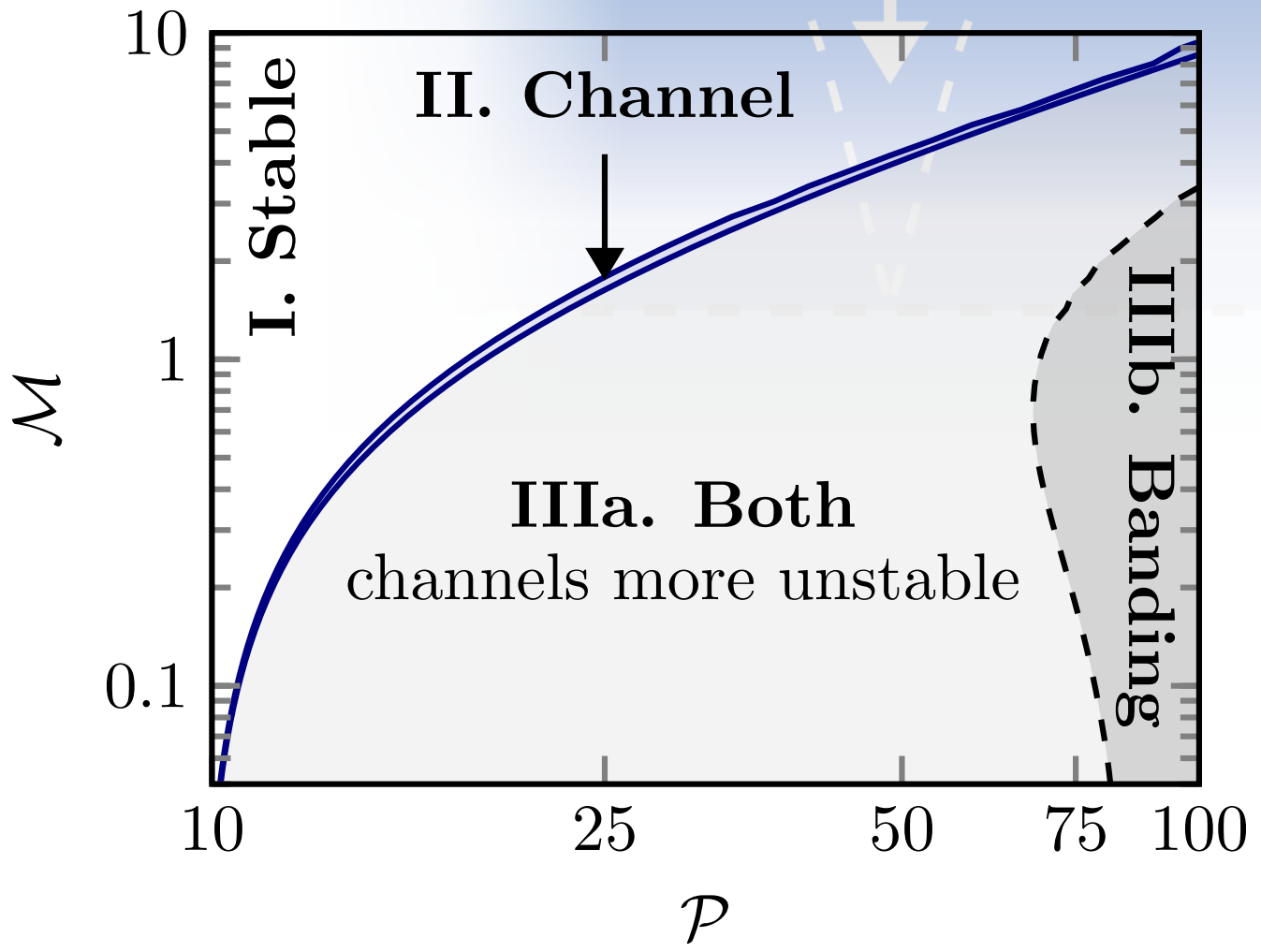
long wavelengths
stabilised by anchoring from the base

short wavelengths
stabilised by diffusion (reswelling)

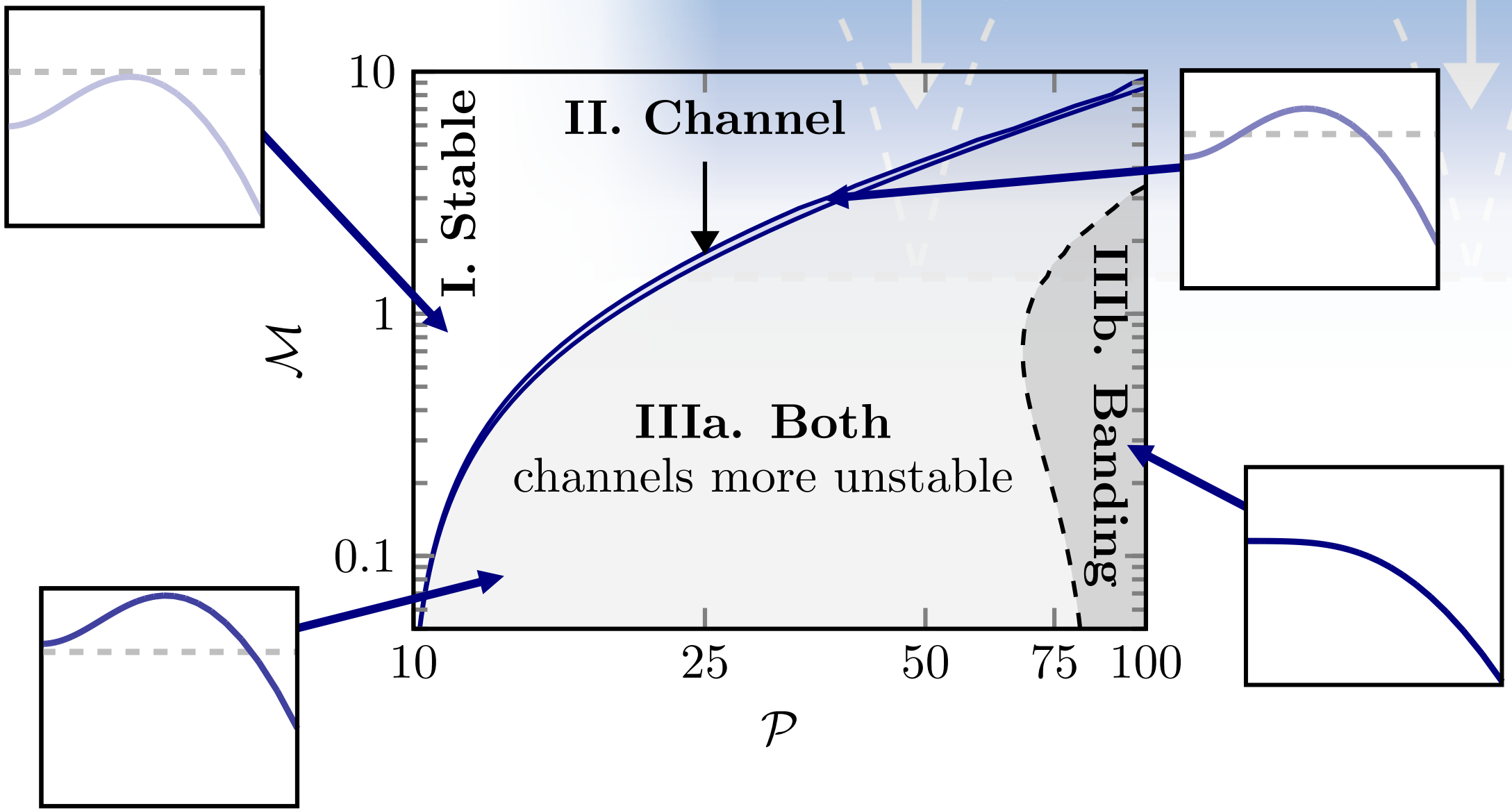
intermediate wavelengths therefore preferred (sometimes…)

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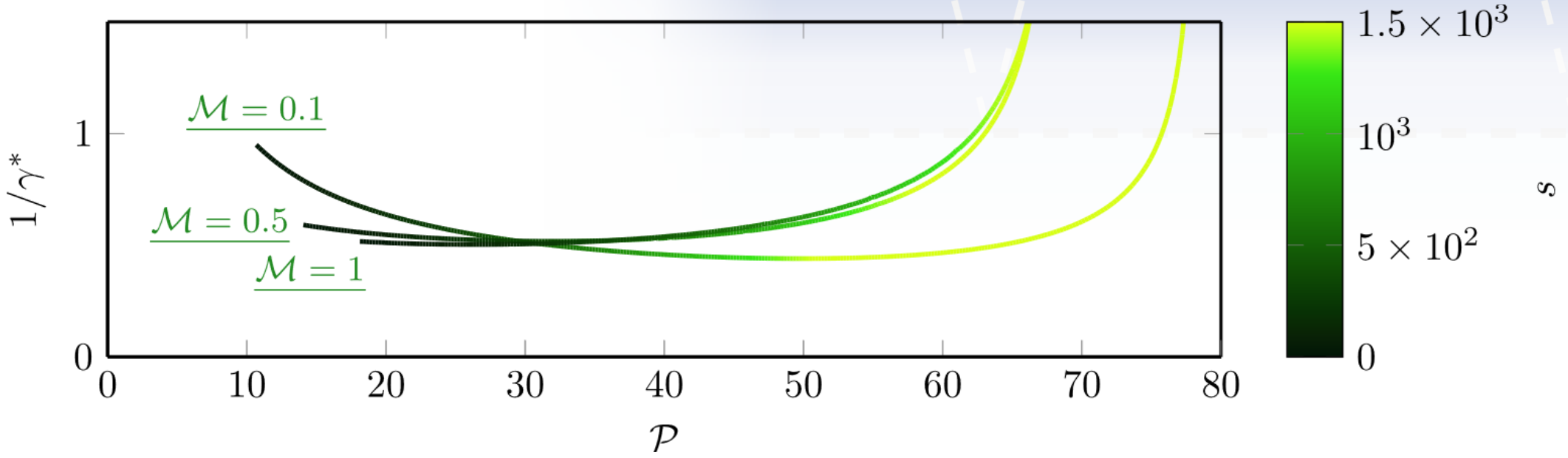
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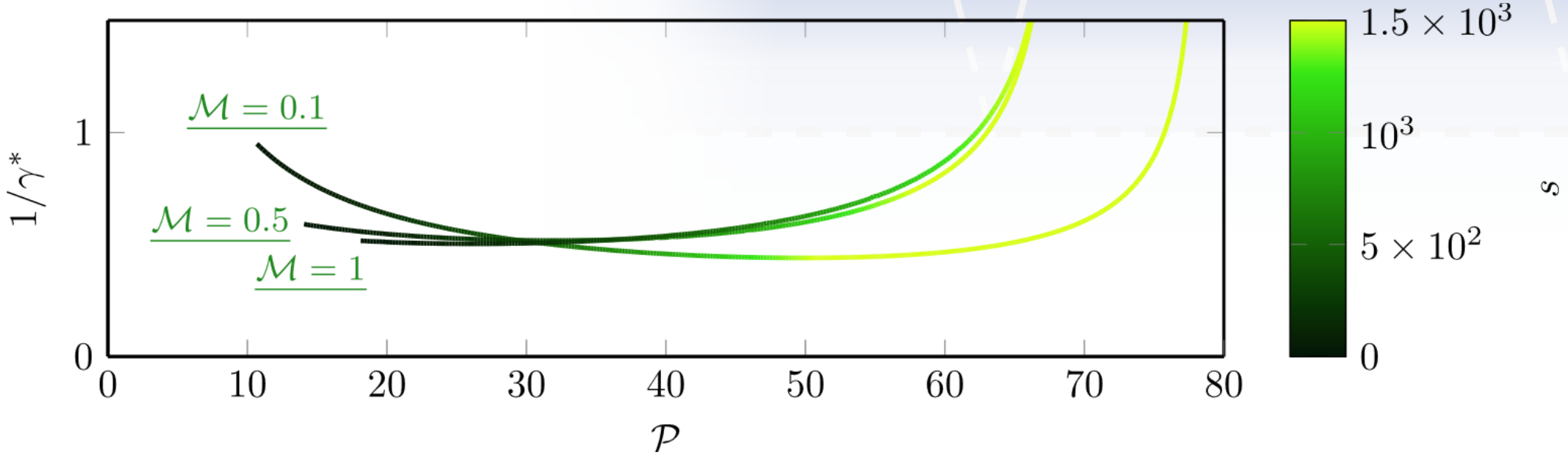
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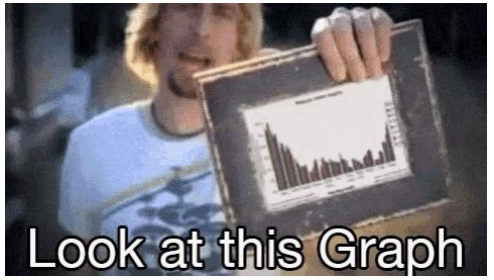


From banding to channelling



From banding to channelling



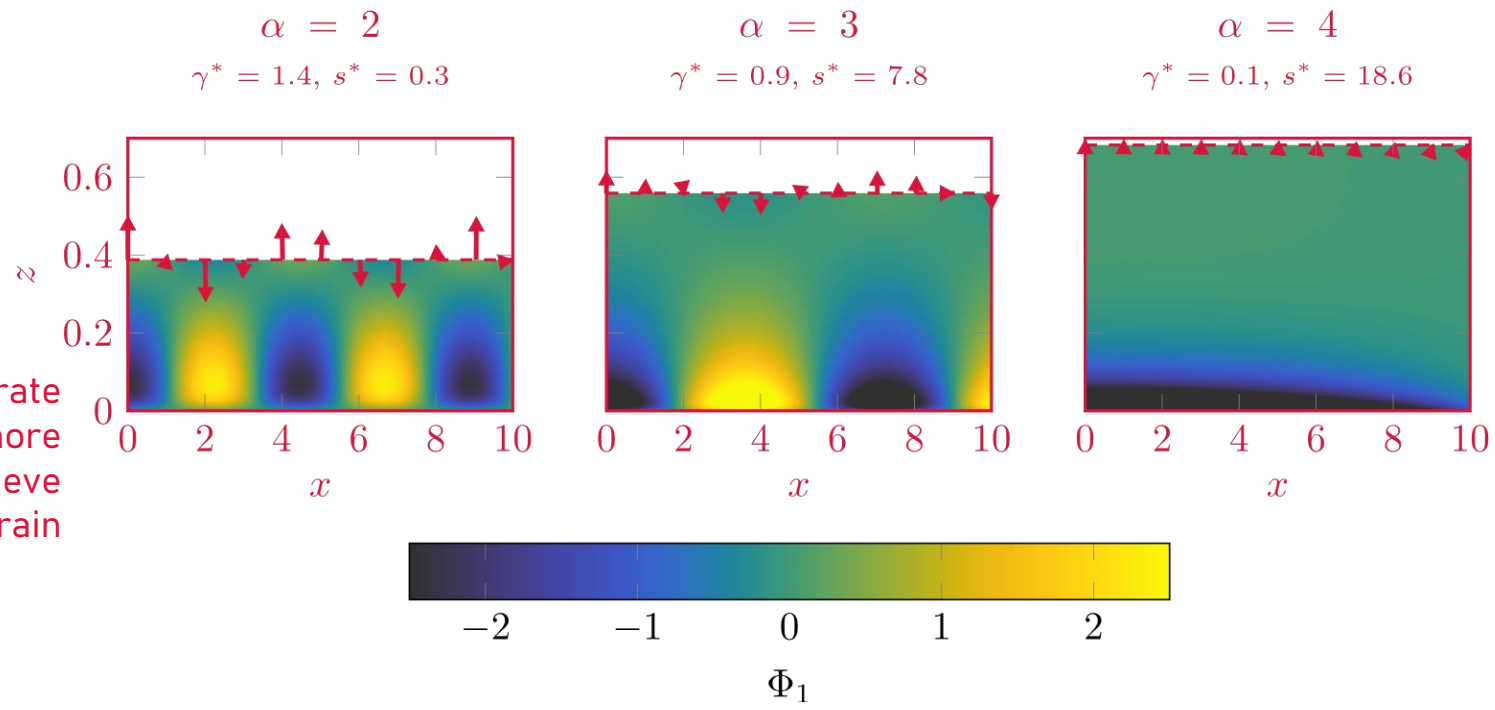


Effect of parameters

Teasing out the individual effect of a parameter value is hard:

- Is the effect based on changing the base state (e.g. large *M* creates an undeformed base state)?
- Is the effect based on the mechanism for growth (e.g. large *M* creates a strong desire to relieve deviatoric strains)?

low shielding; growth rate is slower and looks more like channels to relieve deviatoric strain



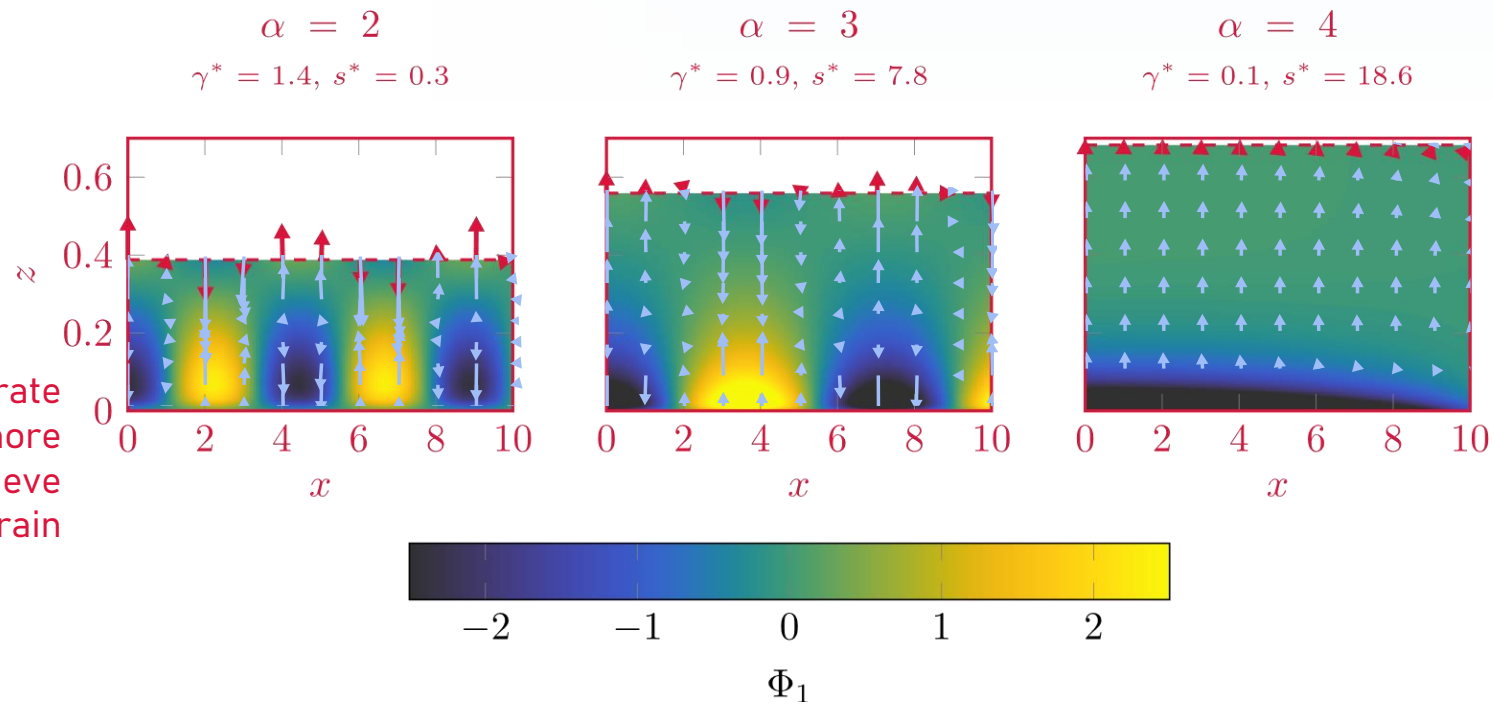
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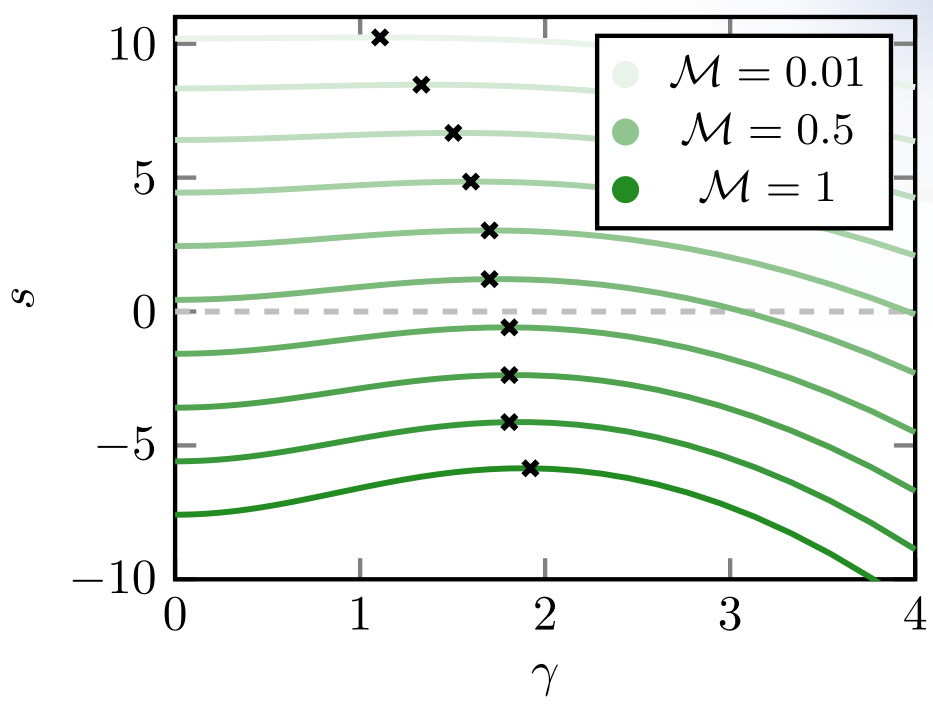


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Effect of parameters

Longer wavelengths for 'softer' gel;
must be a base state effect as we'd expect shear to favour longer 'wrinkles'

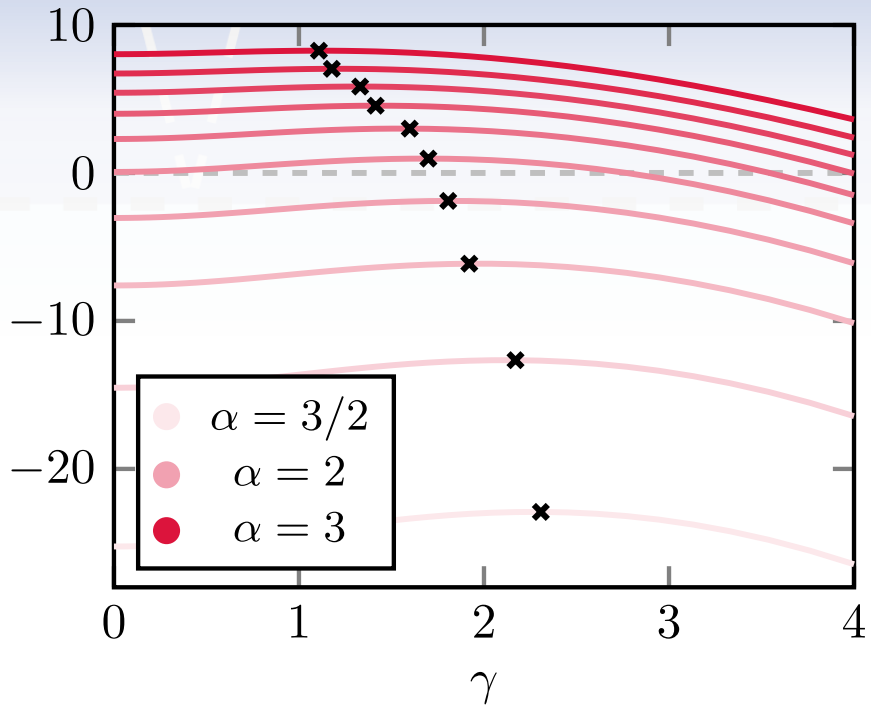
$\alpha = 2$



Lower growth rate for stiffer gel;
less compression in the base state and reduced tendency to form deviatoric channels

Less stability and more horizontal banding as *α* increases;
as seen before

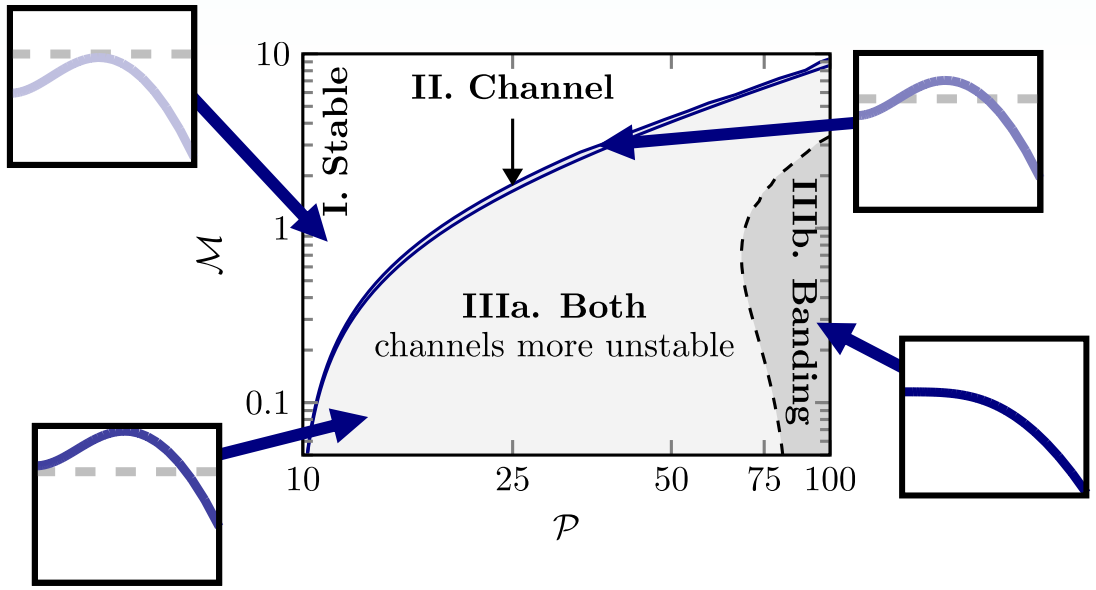
$\mathcal{M} = 0.75$



$\mathcal{P} = 15$

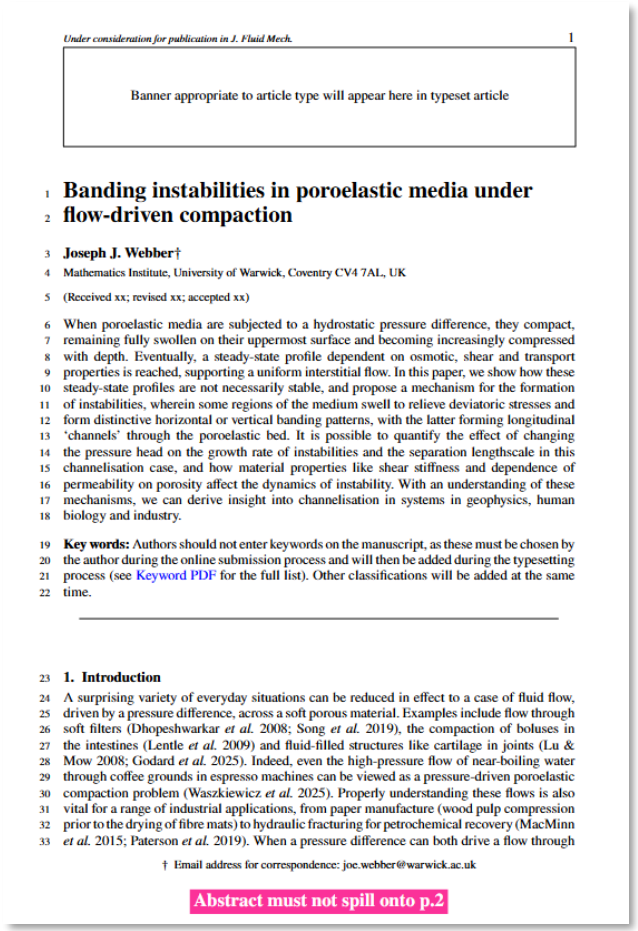
The state of play

- Poroelastic materials appear in industry, nature and biology and their response to pressure differences involves a complex coupling between deformation and flow.
- Compaction depends on the permeability–porosity relationship, stiffness, isotropic elastic properties and the loading pressure. Importantly, compaction is nonuniform.
- **Compaction profiles are unstable** but to two different forms of instability: banding and channelling.
- The effect of stiffness, permeability and osmotic pressure on the instability is difficult to tease out, since these properties modify the base state and the mechanism for instability.



Fluid Dynamics Research Centre seminar
University of Warwick
Wednesday 4th November 2026, 15:00

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Under consideration for publication in J. Fluid Mech. 1

Banner appropriate to article type will appear here in typeset article

Banding instabilities in poroelastic media under flow-driven compaction

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(Received xx; revised xx; accepted xx)

When poroelastic media are subjected to a hydrostatic pressure difference, they compact, remaining fully swollen on their uppermost surface and becoming increasingly compressed with depth. Eventually, a steady-state profile dependent on osmotic, shear and transport properties is reached, supporting a uniform interstitial flow. In this paper, we show how these steady-state profiles are not necessarily stable, and propose a mechanism for the formation of instabilities, wherein some regions of the medium swell to relieve deviatoric stresses and form distinctive horizontal or vertical banding patterns, with the latter forming longitudinal 'channels' through the poroelastic bed. It is possible to quantify the effect of changing the pressure head on the growth rate of instabilities and the separation lengthscale in this channelisation case, and how material properties like shear stiffness and dependence of permeability on porosity affect the dynamics of instability. With an understanding of these mechanisms, we can derive insight into channelisation in systems in geophysics, human biology and industry.

Key words: Authors should not enter keywords on the manuscript, as these must be chosen by the author during the online submission process and will then be added during the typesetting process (see Keyword PDF for the full list). Other classifications will be added at the same time.

1. Introduction

A surprising variety of everyday situations can be reduced in effect to a case of fluid flow, driven by a pressure difference, across a soft porous material. Examples include flow through soft filters (Dhopeshwarkar *et al.* 2008; Song *et al.* 2019), the compaction of boluses in the intestines (Lentle *et al.* 2009) and fluid-filled structures like cartilage in joints (Lu & Mow 2008; Godard *et al.* 2025). Indeed, even the high-pressure flow of near-boiling water through coffee grounds in espresso machines can be viewed as a pressure-driven poroelastic compaction problem (Waszkiewicz *et al.* 2025). Properly understanding these flows is also vital for a range of industrial applications, from paper manufacture (wood pulp compression prior to the drying of fibre mats) to hydraulic fracturing for petrochemical recovery (MacMinn *et al.* 2015; Paterson *et al.* 2019). When a pressure difference can both drive a flow through

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Abstract must not spill onto p.2

More details available in *Banding instabilities in poroelastic media under flow-driven compaction*

Draft manuscript; happy to share for comments!

With thanks to:




Tom Montenegro-Johnson, Ellen Jolley
Warwick




Grae Worster, Ducan Hewitt
Cambridge

UNIVERSITY OF WARWICK