Session 8, Category 04: *Poroelastic flows*
EFDC2 Dublin, Thursday 28th August 2025, 15:00

Poromechanical modelling of pumping with responsive hydrogels

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based on
Webber & Montenegro-Johnson ‘Poromechanical modelling of thermo-responsive hydrogel pumps’
J. Fluid Mech. 1009 (2025)

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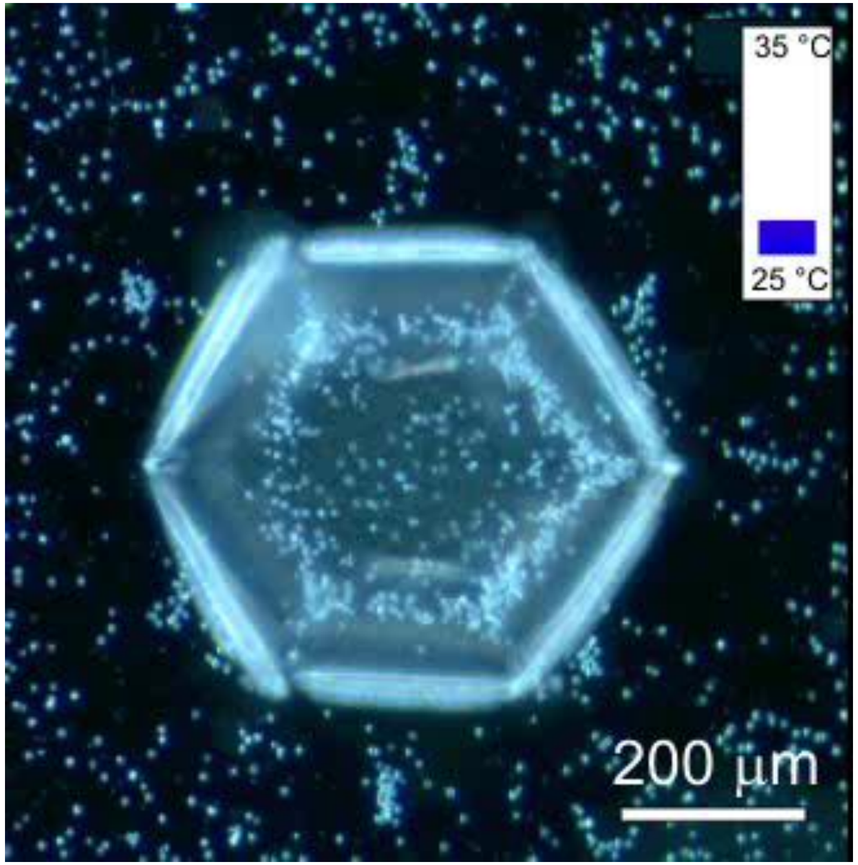
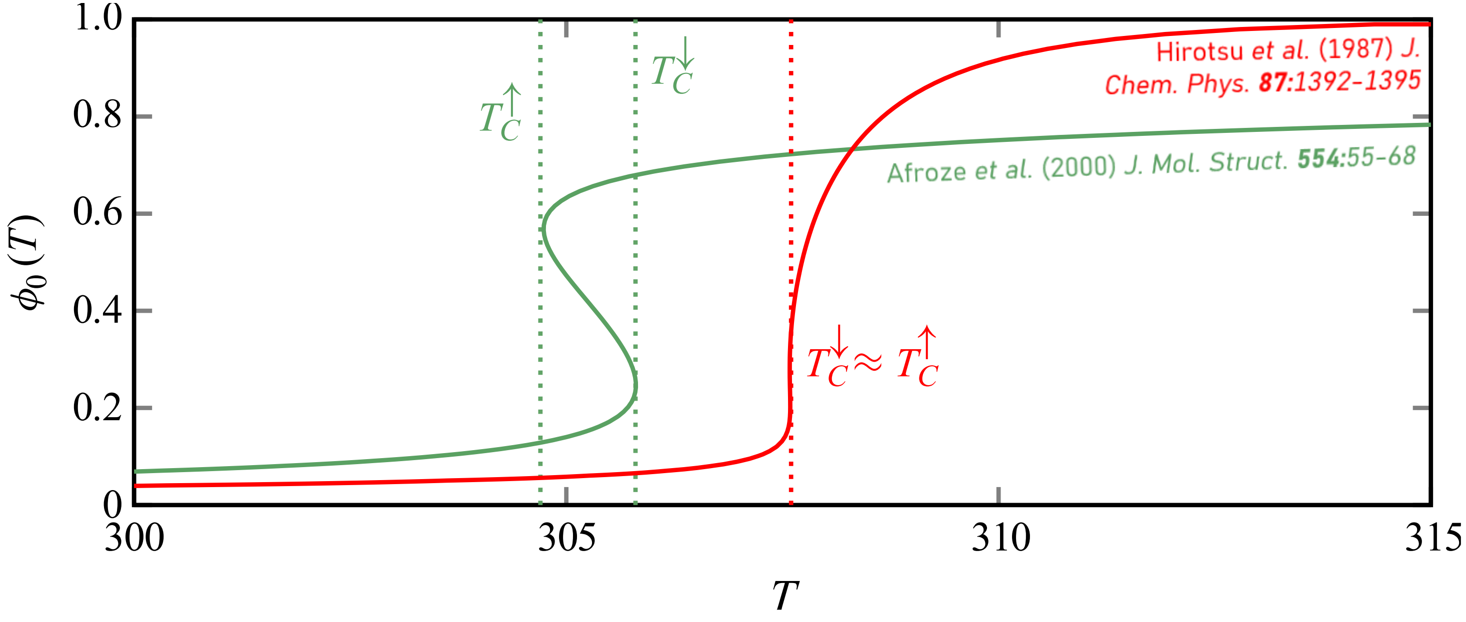
Thermo-responsive hydrogels

Some gels like poly(N-isopropylacrylamide) (pNIPAM) undergo a transition at a critical temperature called the Lower Critical Solution Temperature (LCST)



Stoychev *et al.* (2011) *Soft Matter* **7**:3277–3279

Butler & Montenegro-Johnson (2022) *J. Fluid Mech.* **947**:A11



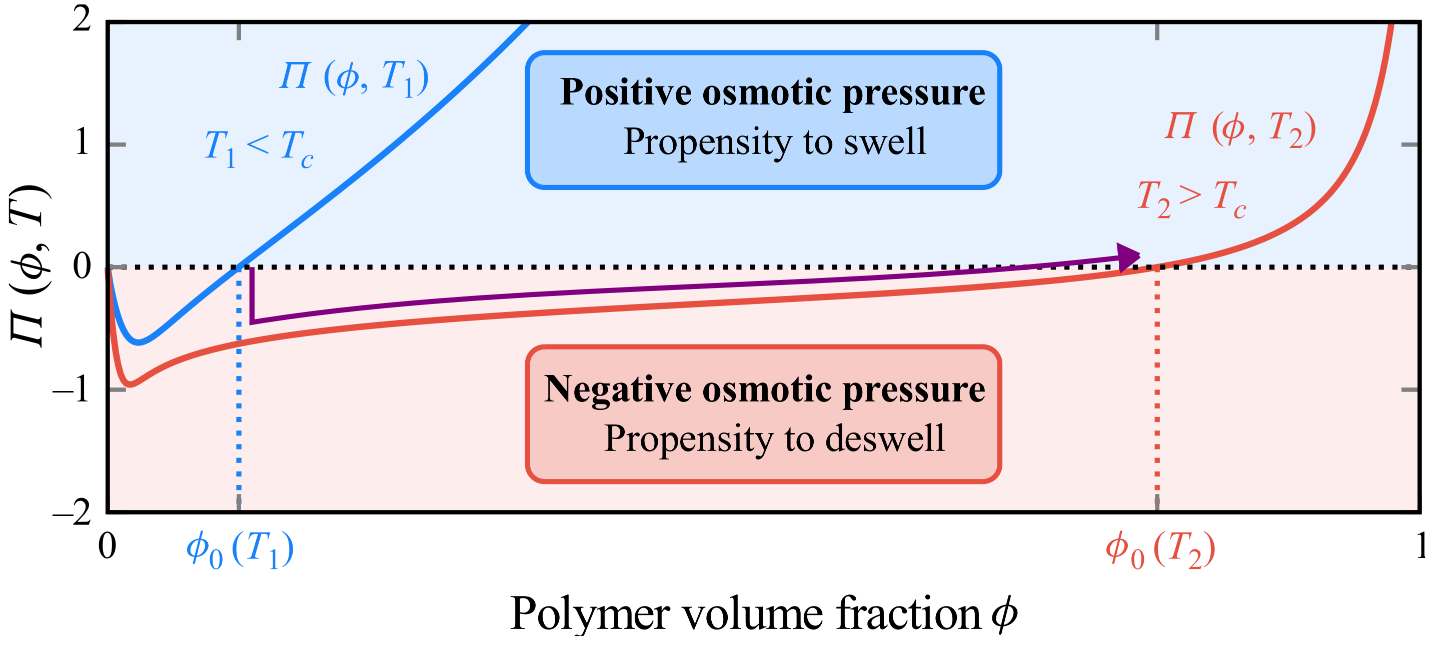
$$\phi_0 = \begin{cases} \phi_{00} & T \le T_C \\ \phi_{0\infty} & T > T_C \end{cases}$$

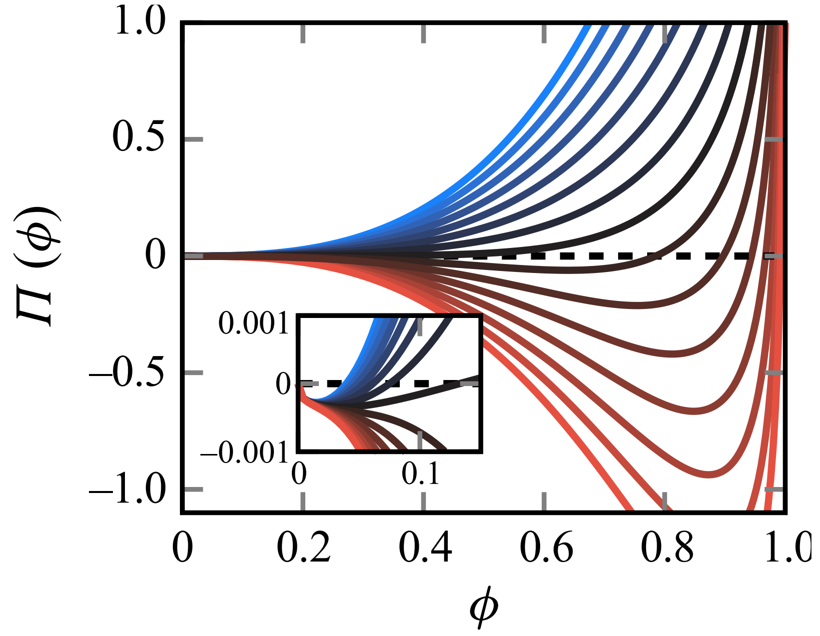
Φ_0 represents the 'rest state' polymer volume fraction of the gel

A generalised osmotic pressure...

$$\mathcal{W} = \frac{k_B T}{2\Omega_p}\left[\mathrm{tr}\left(\mathbf{F_d}\,\mathbf{F_d^T}\right) - 3 + 2\log\phi\right] + \frac{k_B T}{\Omega_f}\left[\frac{1-\phi}{\phi}\log(1-\phi) + \chi(\phi, T)(1-\phi)\right]$$

Flory-Rehner model for free energy density





Phenomenologically, what do we see here? Sudden change in equilibrium, and linearise around it:

$$\Pi(\phi) = \Pi_0 \frac{\phi - \phi_0(T)}{\phi_0(T)}$$

…viewed through a responsive LENS

osmotic pressure
the affinity of polymer scaffold for water

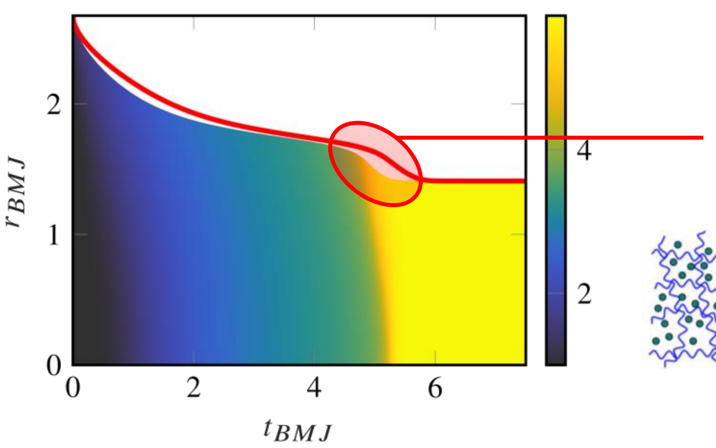
shear modulus
the resistance to shearing deformation

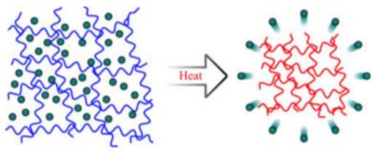
permeability
the resistance to viscous flow through the scaffold

$$\boldsymbol{\sigma} = -\left[p + \Pi(\phi)\right]\boldsymbol{I} + 2\mu_s(\phi)\boldsymbol{\epsilon} \qquad \boldsymbol{u} = -\frac{k(\phi)}{\mu_l}\boldsymbol{\nabla} p$$

(pore pressure)

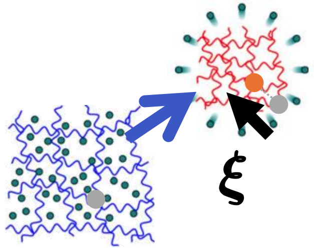
Φ (normalised polymer fraction)





$$\frac{\partial\phi}{\partial t} + \boldsymbol{q}\cdot\boldsymbol{\nabla}\phi = \boldsymbol{\nabla}\cdot\left\{\frac{k(\phi)}{\mu_l}\left[\phi\frac{\partial\Pi}{\partial\phi} + \frac{4\mu_s(\phi)}{3}\left(\frac{\phi}{\phi_0}\right)^{1/3}\right]\boldsymbol{\nabla}\phi\right\}$$

$$\boldsymbol{u} = (1-\phi)(\boldsymbol{u_w} - \boldsymbol{u_p})$$
$$\boldsymbol{q} = (1-\phi)\boldsymbol{u_w} + \phi\boldsymbol{u_p}$$



$$\nabla^4\boldsymbol{\xi} = -3\boldsymbol{\nabla}\nabla^2\left(\frac{\phi}{\phi_0}\right)^{1/3}$$

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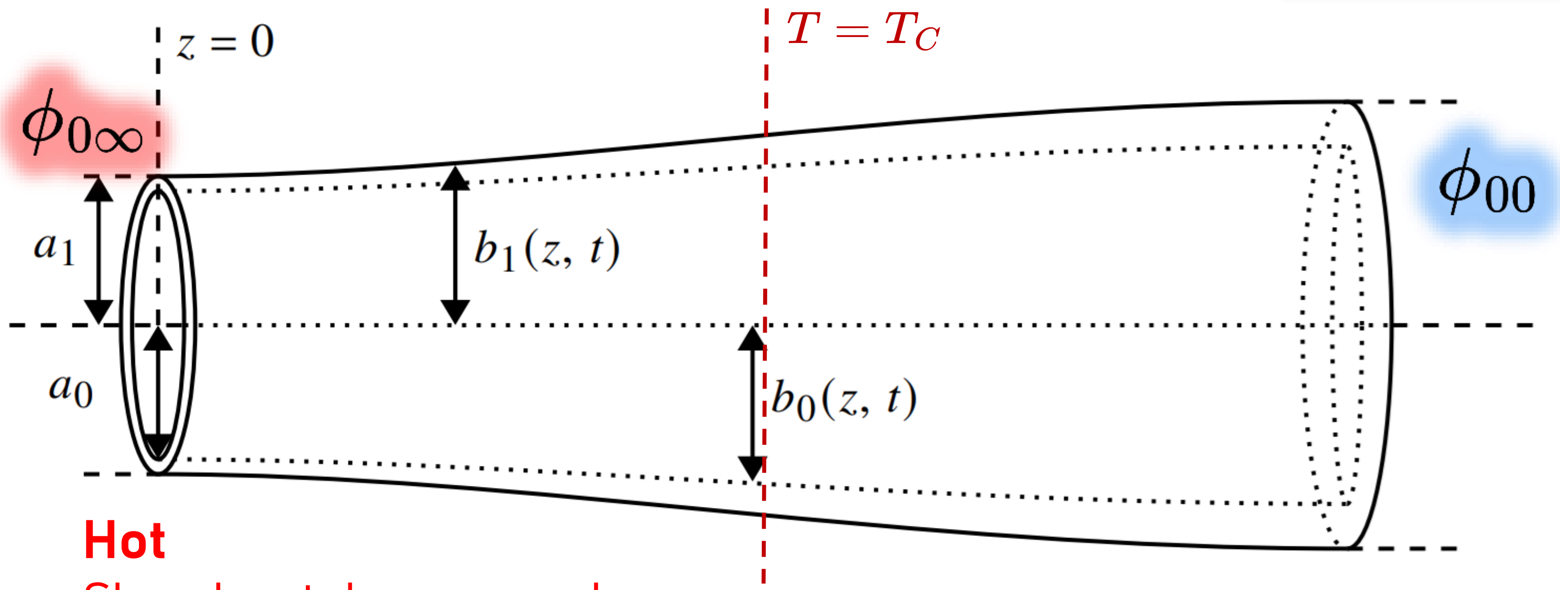
Tubular gel pumps

Assume slenderness $\varepsilon = a_1/L \ll 1$ and $\phi = \phi_1(z, t) + \varepsilon^2 \phi_2(r; z, t)$



Hot
Shrunken tube, narrow lumen
$T = T_C + \Delta T$

Cold $T \to T_C - \Delta T$
Relaxed tube, wide lumen

$$\frac{\partial \phi}{\partial t} + \boldsymbol{q} \cdot \boldsymbol{\nabla} \phi = \frac{1}{r} \frac{\partial}{\partial r}\left[r D(\phi, T) \frac{\partial \phi}{\partial r} \right] + \frac{\partial}{\partial z}\left[D(\phi, T) \frac{\partial \phi}{\partial z} \right]$$

$$D(\phi, T) = \frac{k}{\mu_l}\left[\frac{\Pi_0(T)\phi}{\phi_0(T)} + \frac{4\mu_s}{3}\left(\frac{\phi}{\phi_{00}} \right)^{1/3} \right]$$

with $\phi = \phi_0(T)$ on boundaries

- How does the heat pulse spread?
- Can we quantify $\boldsymbol{q}$?
- How does the shape of the tube change?
- What flows result from reconfiguration?

Decoupling the thermal and poroelastic problems

external heat supply

viscous heat generation

$$\frac{\partial T}{\partial t} + \boldsymbol{q}\cdot\boldsymbol{\nabla}T = \frac{R}{\rho c} + \kappa\nabla^2 T + \frac{k(\phi)}{\rho c \mu_l}|\boldsymbol{\nabla}p|^2 + \frac{1}{\phi}\left(\frac{\Pi(\phi)}{\rho c} + T\right)\frac{\mathrm{d}\phi}{\mathrm{d}t}$$

advection
as gel deforms

diffusion

swelling/drying energy changes
cf. latent heat

reconfiguration is ‘slow’ compared to diffusion of heat (*Le* ≫ *1*) ⟶ ***$\partial T/\partial t \approx \kappa\, \partial^2 T/\partial z^2$***

thermal diffusivity of water is similar to that of gel ⟶ ***solve in single domain***

$$\boxed{T - T_C = \Delta T\left[2\,\mathrm{erfc}\left(\frac{z}{2\sqrt{\kappa t}}\right) - 1\right]}$$

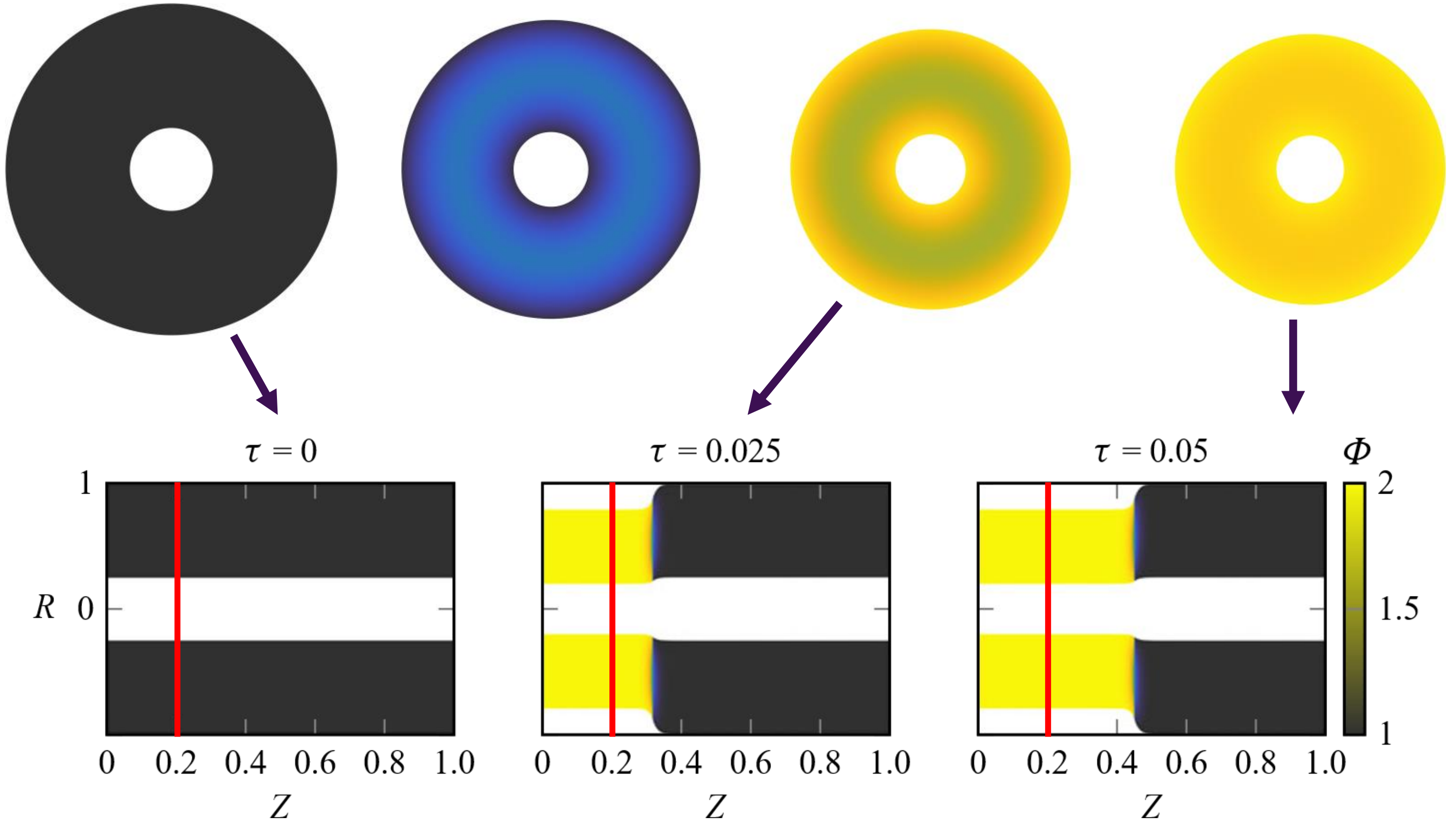
gel deswells behind thermal front, remains swollen in front

$$\phi_0 = \begin{cases} \phi_{00} & z > 2\,\mathrm{erfc}^{-1}(1/2)\sqrt{\kappa t} \\ \phi_{0\infty} & z \leq 2\,\mathrm{erfc}^{-1}(1/2)\sqrt{\kappa t} \end{cases}$$

Deformation of the tube

drying begins slightly ahead of front (along axis)

drying behind front occurs radially outwards



locally-isotropic deformation gives

$$\frac{\xi}{r} \approx \frac{\partial \xi}{\partial r} \approx \frac{\partial \eta}{\partial z} \approx 1 - \left(\frac{\phi}{\phi_{00}}\right)^{1/3}$$

so

$$\frac{b_0}{a_0} \approx \frac{b_1}{a_1} \approx \left(\frac{\phi}{\phi_{00}}\right)^{-1/3}$$

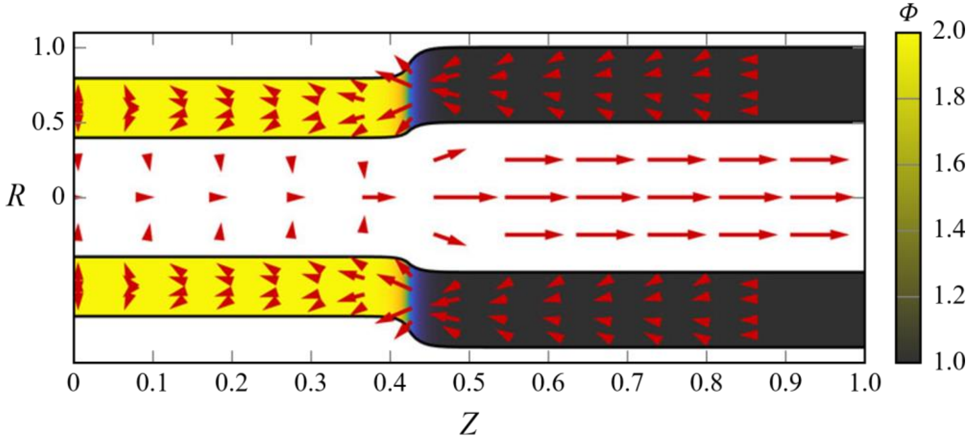
then, separation of variables implies that

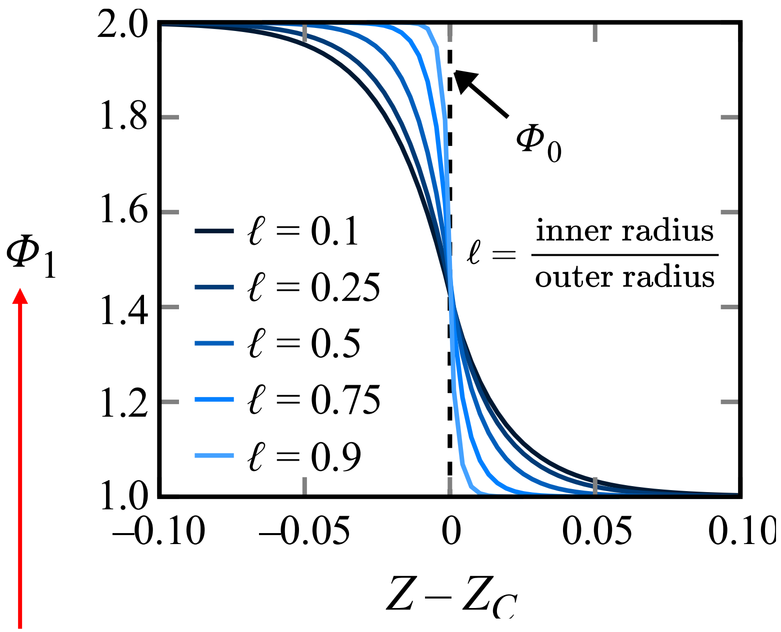
$$\phi_2 = \frac{4[\phi_0(T) - \phi_1]}{\varepsilon^2(1-\ell)^2}\left(\frac{\phi_1}{\phi_{00}}\right)^{2/3}\left[\frac{r}{a_1} - \frac{1+\ell}{2}\left(\frac{\phi_1}{\phi_{00}}\right)^{-1/3}\right]^2$$

deviations from equilibrium polymer fraction must be small

Flows through and in the tube

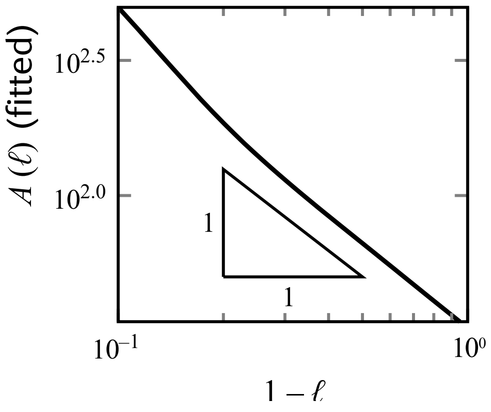
- **Radial fluxes** from walls into surroundings $u_r \propto \partial\phi/\partial r$
- **Axial fluxes** through the gel from more swollen to less swollen $u_z \propto \partial\phi/\partial z$
- **Axial fluxes** in lumen: tube collapses and 'squeezes'





interior (middle of tube wall) polymer fraction (scaled)

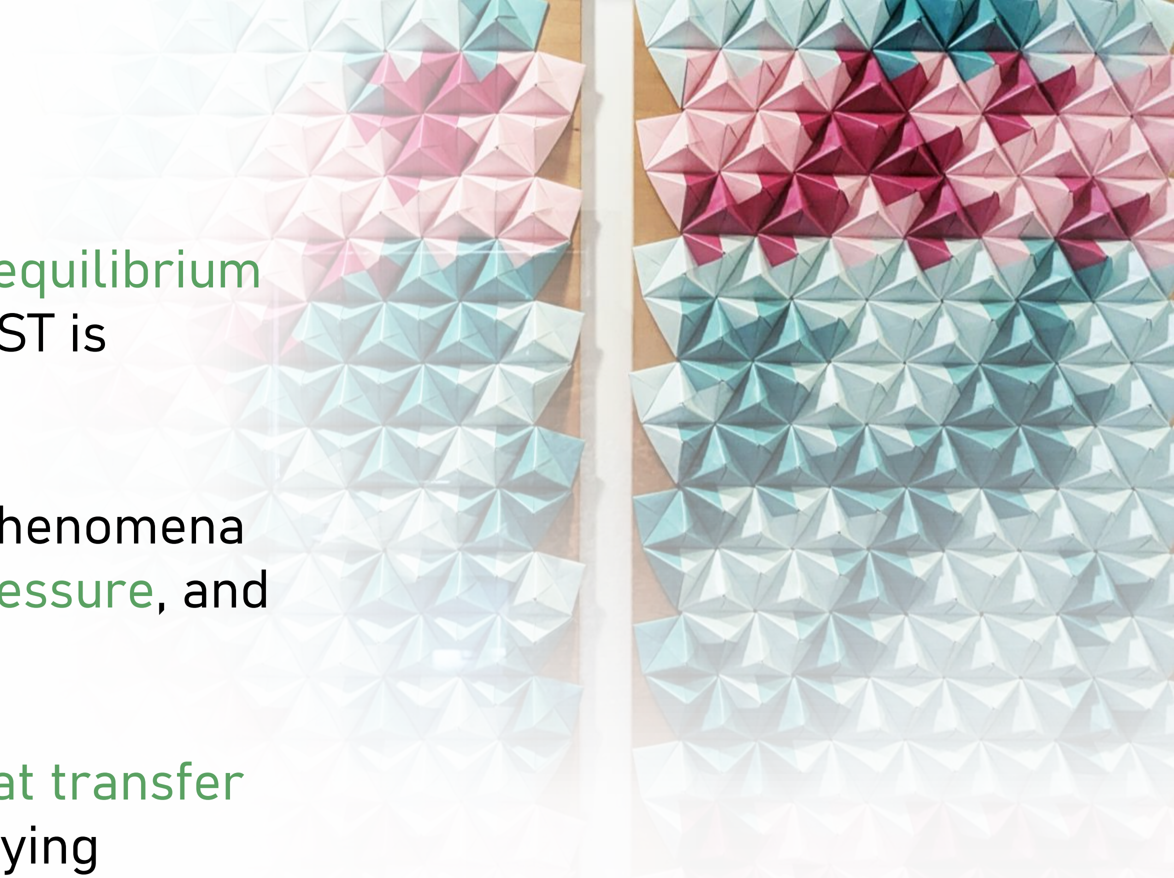
$$\Phi_1 \approx \Phi_\infty - \frac{\Phi_\infty - 1}{2}\{1 + \tanh\left[A(\ell)(Z - Z_C)\right]\}$$

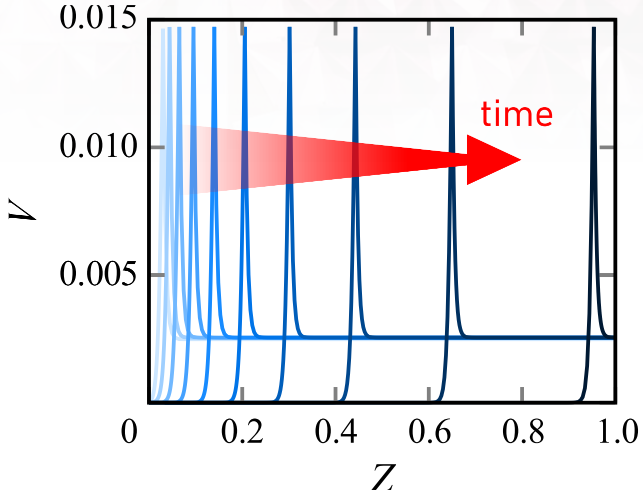


Rate of collapse depends on the thickness of the tube walls, which affects the magnitude of the lumen pulse

Conclusions and applications

- Thermo-responsive gels can be modelled using an equilibrium polymer fraction that changes sharply when the LCST is crossed
- This allows us to capture swelling and deswelling phenomena without knowing the precise form of the osmotic pressure, and can replicate more complicated models effectively
- We write down a straightforward expression for heat transfer in gels including contributions from swelling and drying
- We can build models for microfluidic ‘pumps’ that transport fluid as a pulse of heat spreads out in time, quantifying all the deswelling fluxes
- Geometry affects the response times and can be tuned to give required fluxes (pictured)





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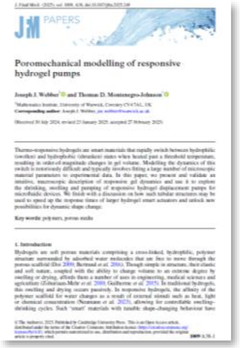
with thanks to

Tom Montenegro-Johnson
Warwick

Grae Worster
Cambridge



more details can be found in

Webber, J. J. & Montenegro-Johnson, T. D.
Poromechanical modelling of responsive hydrogel pumps
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